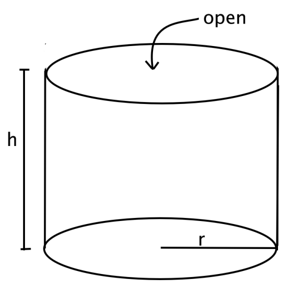
MATH 1210 Lab 9 (10/19/2017)

Student #:______________________ Instructor & Section:______________________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed, unless stated otherwise.

1. A cylindrical can with an **open top** is to be made out of sheet metal and is to have a volume of 8π in^3
   (a) Let A denote the amount of metal used to make the can (the surface area of the can). Find a formula for A in terms of r and h.
   (b) Find the relationship between r and h resulting from the fixed volume.
   (c) Use (b) above to write A as a function of r alone.
   (d) Find the value of r that minimizes A, the amount of metal used to make the can.



Solution:

(a) $A = \pi r^2 + 2\pi r h$.

(b) $8\pi = \pi r^2 h$.

(c) Using (b), $h = \frac{8}{r^2}$, and so $A = \pi r^2 + 2\pi r(\frac{8}{r^2}) = \pi r^2 + \frac{16\pi}{r}$.

(d) Find critical points:

$$0 = A'(r) = 2\pi r - \frac{16\pi}{r^2} \Rightarrow \frac{16\pi}{r^2} = 2\pi r \Rightarrow r^3 = 8 \Rightarrow r = 2 \text{ inches}$$

Note that by the First Derivative Test (or the Second, for that matter), $r = 2$ is a local minimum.

2. A symphony orchestra plays a yearly concert at a large outdoor stage. The orchestra wants to maximize the profit that it makes from the concert. Last year, the ticket price was \$25 and 600 people attended. The year before, 400 people attended when the ticket price was \$30. It costs $\$10,000$ to rent the venue and pay the players. Assuming that the demand (number of tickets sold) is a linear function of the price of the tickets, what should the price of the tickets be in order to maximize the profit and what is this maximum profit?

Solution: Let $n(p)$ denote the number of tickets sold if priced at p dollars. Then we know that $n(25) = 600$ and $n(30) = 400$. Assuming n is linear, we can find the equation given the two points: $n(p) = -40p + 1600$. The profit obtained if the tickets are priced at p dollars is therefore

$$P(p) = pn(p) - 10,000 = -40p^2 + 1600p - 10000.$$

We look for critical points of P:

$$0 = P'(p) = -80p + 1600 \Rightarrow p = 20$$

Note that $p = 20$ is in fact a local max (by either derivative test). So the tickets should be priced at $p = 20$ dollars, in which case the profit is $P(20) = 6000$ dollars.

3. Answer the following questions about the function,
$f(x) = \dfrac{x}{(x-1)^2}$ and use the information to draw the graph.

(a) Does $f(x)$ have any vertical asymptotes?

Answer:________________________

(b) Compute $f'(x)$.

Answer:________________________

(c) Find the critical point(s) of $f(x)$.

(d) Find where $f(x)$ is increasing and decreasing.

Critical Points:________________________
Increasing:________________________
Decreasing:________________________

(e) Compute $f''(x)$.

Answer:________________________

(f) Find any inflection points and where $f(x)$ is concave up and down.

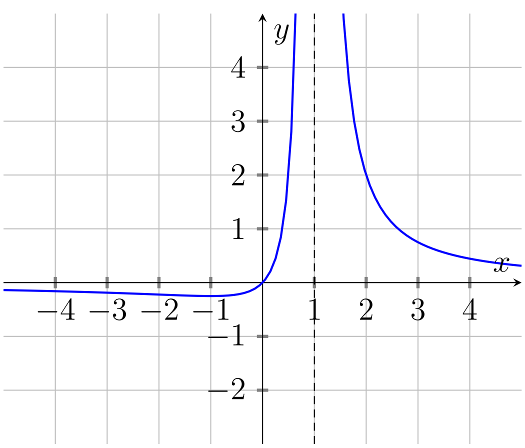
Inflection Points:________________________

Concave up:________________________

Concave down:________________________

(g) Finally, evaluate the function at the following points and sketch a graph.

$f(-3) =$_____ $f(-1) =$_____ $f(0) =$_____ $f(2) =$_____



Solution:

(a) Vertical Asymptote: $x = 1$

(b) $f'(x) = \dfrac{(x-1)^2 - 2x(x-1)}{(x-1)^4} = \dfrac{-(x+1)}{(x-1)^3}$

(c) $f'(x) = 0 \Rightarrow \dfrac{-(x+1)}{(x-1)^3} = 0$. We have a critical point at $x = -1$. Note, $x = 1$ is not a critical point because f is not defined there. However, the vertical asymptote $x = 1$, is on the sign line determining where f is increasing and decreasing.

(d) $f(x)$ is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1) \cup (1, \infty)$

(e) $f''(x) = \dfrac{(x-1)^3(-1) - (-(x-1))(3(x-1)^2)}{(x-1)^6} = \dfrac{-(x-1) + 3(x+1)}{(x-1)^4}$

$= \dfrac{2x+4}{(x-1)^4} = \dfrac{2(x+2)}{(x-1)^4}$

(f) There is an inflection point when $x = -2$ so, $(-2, -\frac{2}{9})$. $f(x)$ is concave up on $(-2, 1) \cup (1, \infty)$ and concave down on $(-\infty, -2)$.

(g) $f(-3) = -\frac{3}{16}$, $f(-1) = -\frac{1}{4}$, $f(0) = 0$, $f(2) = 2$