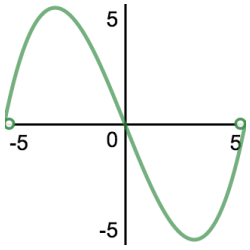
MATH 1210 Lab 8 (10/5/2017)

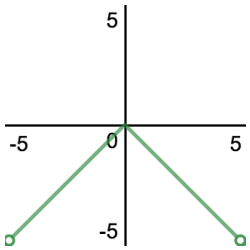
Student #:_____ Instructor & Section:_____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed, unless stated otherwise.

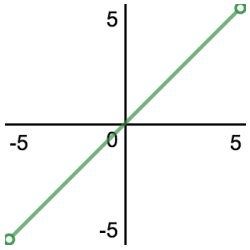
1. Sketch an example of a function satisfying the following:
   (a) A function defined on $(-5, 5)$ with both a maximum and a minimum.
   Note: There are multiple correct graphs. Below is just one possible set of answers.



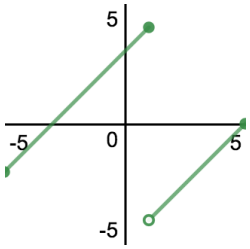
   (b) A function defined on $(-5, 5)$ with a maximum but no minimum.



   (c) A function defined on $(-5, 5)$ with neither a maximum nor minimum.



   (d) A function defined on $[-5, 5]$ with a maximum but no minimum.



2. Find the critical points of the following functions on the entire real number line.

(a) $p(x) = x^4 - 5x^3 + 9$

Answer:_____

(b) $q(x) = x^{\frac{2}{5}}(x - 35)$

Answer:_____

(c) $r(x) = \cos x - \sin x$

Answer:_____

Solution:

(a) $p'(x) = 4x^3 - 15x^2 = x^2(4x - 15)$. Setting this equal to zero we see that we have a critical point at $x = 0$ and $x = \frac{15}{4}$.

(b) $q'(x) = 2/5x^{-3/5}(x - 4) + x^{2/5} = \frac{\frac{7}{5}x - 14}{x^{3/5}}$, which gives critical points at $x = 0$ and $x = 10$.

(c) $r'(x) = -\sin x - \cos x$. Solving $\sin x = -\cos x$ we get $x = 3\pi/4 + \pi k$ for $k \in \mathbb{Z}$.

3. Identify the critical points and find the extreme values on the given interval.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[0, 3]$

Answer:_____

(b) $g(x) = -2\cos x - x^2$ on $[-\frac{\pi}{2}, \pi]$

Answer:_____

(c) $h(x) = |x^2 - 6x + 8|$ on $[0, 4]$

Answer:_____

Solution:

(a) $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$ Solving for $f'(x) = 0$ we find $x = 2, -1$. However, $x = -1$ is not on the interval specified. So the critical points are $x = 0, 2, 3$. $f(0) = 1$. $f(2) = -19$. $f(3) = -8$. So the max value is 1, while the min value is -19.

(b) $g'(x) = 2\sin x - 2x$ and so we will have a critical point if $\sin x = x$. You can see graphically that this has only the solution $x = 0$. So the critical points are $x = -\frac{\pi}{2}, 0, \pi$. $g(-\frac{\pi}{2}) = -\frac{\pi^2}{4}$, $g(0) = -2$, and $g(\pi) = 2 - \pi^2$. The max value is -2 and the min value is $2 - \pi^2$.

(c) It might be helpful to produce a graph of the function. The critical points will be where the derivative of $x^2 - 6x + 8$ is zero and where $x^2 - 6x + 8 = 0$ (since the derivative will be undefined there). Since $D_x(x^2 - 6x + 8) = 2x - 6$ and $x^2 - 6x + 8 = (x-2)(x-4)$, our critical points are $x = 0, 2, 3, 4$. We then evaluate: $h(0) = 8$, $h(2) = 0$, $h(3) = 1$, $h(4) = 0$. The max value is 8 and the min value is 0.

4. Suppose $f(x)$ and $g(x)$ are differentiable. Show that if both $f(x)$ and $g(x)$ are increasing, then $f(g(x))$ is also increasing. If both $f(x)$ and $g(x)$ are decreasing, is it true that $f(g(x))$ is decreasing too?

Solution: If $f(x)$ and $g(x)$ are increasing, then $f'(x) > 0$ and $g'(x) > 0$ at all values of x. By the chain rule,

$$D_x(f(g(x)) = f'(g(x))g'(x) > 0$$

since it is the product of two positive numbers. Hence $f(g(x))$ is increasing.

If both $f(x)$ and $g(x)$ are decreasing, then $f'(x) < 0$ and $g'(x) < 0$ for all values of x. Again the chain rule says that

$$D_x(f(g(x)) = f'(g(x))g'(x) > 0$$

since the product of two negative numbers is positive. So $f(g(x))$ is in fact increasing too.