MATH 1210 Lab 3 (9/07/2017)

Student #:_____ Instructor & Section:_____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed, unless stated otherwise.

1. Consider the function

$$f(x) = \frac{x^2+1}{x^3-1}$$

Compute the following. Answers may be numbers, $+\infty$, or $-\infty$.

(a) $\lim_{x\to\infty} f(x) =$

Answer:_____

(b) $\lim_{x\to-\infty} f(x) =$

Answer:_____

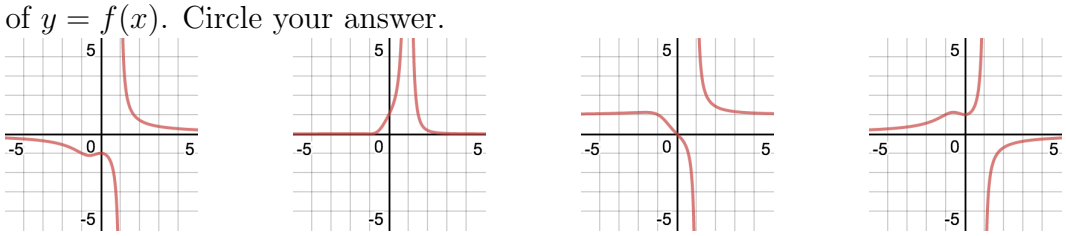
(c) $\lim_{x\to1^-} f(x) =$

Answer:_____

(d) $\lim_{x\to1^+} f(x) =$

Answer:_____

(e) Use the information you found above to determine which graph below is the graph of $y = f(x)$. Circle your answer.



Solution:

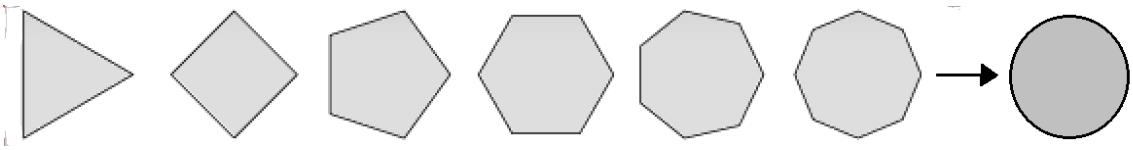
(a) 0

(b) 0

(c) $-\infty$

(d) ∞

(e) The first one.



2. In this problem, we'll use limits and the idea that regular polygons give better and better approximations to a circle as the number of sides increase.

   (a) Suppose an n-sided regular polygon is inscribed in a circle of radius r. The polygon is composed of n triangles (see Figure 1 below). If we zoom in on one of the triangles (Figure 2 below), what is the measure (in radians) of the angle labeled θ? Keep n arbitrary.

   (b) Find the height of the triangle, labeled h in Figure 2, and use this to find the area of ΔPQR.

   (c) Find the area of the inscribed polygon. Remember, it is made of of n triangles.

   (d) Take the limit as $n \to \infty$ of your answer in (c) and show that you get πr^2, the area of the circle of radius r. **Hint:** It may be helpful to make a change of variables $\theta = \frac{2\pi}{n}$ and let $\theta \to 0$ instead.

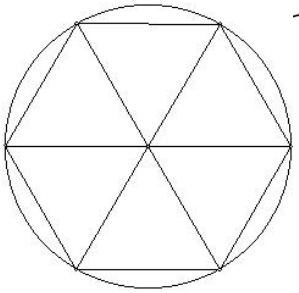


Figure 1

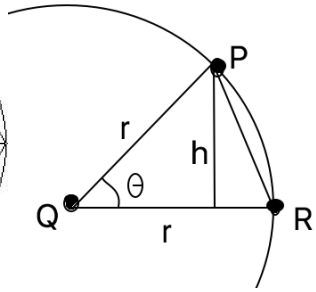


Figure 2

(e) Now compute the length of the line segment $\overline{PR}$ as a function of n. **Hint:** You'll have to remember the Law of Cosines.

(f) What is the perimeter of the inscribed polygon? Again, remember that it is made up of n triangles.

(g) Now take the limit as $n \to \infty$ of your answer in part (e) and show that you get $2\pi r$, the circumference of the circle of radius r. **Hint:** Again, the change of variables $\theta = \frac{2\pi}{n}$ might be helpful and you'll also need to use the fact that $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$.

Solution:

(a) $\theta = \dfrac{2\pi}{n}$

(b) $h = r\sin\theta = r\sin\left(\dfrac{2\pi}{n}\right)$ and so $A = \dfrac{1}{2}rh = \dfrac{r^2}{2}\sin\left(\dfrac{2\pi}{n}\right)$.

(c) $\dfrac{r^2 n}{2}\sin\left(\dfrac{2\pi}{n}\right)$.

(d) If $\theta = \dfrac{2\pi}{n}, n = \dfrac{2\pi}{\theta}$

and,

$$\frac{r^2 n}{2}\sin\left(\frac{2\pi}{n}\right) = \frac{2\pi r^2}{2\theta}\sin\theta = (\pi r^2)\frac{\sin\theta}{\theta}.$$

Since,

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} \to 1$$

we have $\pi r^2 \cdot 1 = \pi r^2$.

(e) The Law of Cosines gives that

$$|\overline{PR}|^2 = 2r^2 - 2r^2\cos\left(\frac{2\pi}{n}\right),$$

and so

$$|\overline{PR}| = \sqrt{2}r\sqrt{1-\cos\left(\frac{2\pi}{n}\right)}.$$

(f) $\sqrt{2}rn\sqrt{1-\cos\left(\dfrac{2\pi}{n}\right)}$.

(g) If $\theta = \dfrac{2\pi}{n}$, then

$$\sqrt{2}rn\sqrt{1-\cos\left(\frac{2\pi}{n}\right)} = \sqrt{2}r\left(\frac{2\pi}{\theta}\right)\sqrt{1-\cos\theta} = 2\sqrt{2}\pi r\sqrt{\frac{1-\cos\theta}{\theta^2}}.$$

Then

$$\lim_{\theta\to 0} 2\sqrt{2}\pi r\sqrt{\frac{1-\cos\theta}{\theta^2}} = 2\sqrt{2}\pi r\sqrt{\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}} = 2\sqrt{2}\pi r\sqrt{\frac{1}{2}} = 2\pi r.$$

3. In this problem, you will see that you can't reliably do arithmetic with infinities. For each part, find two functions $f(x)$ and $g(x)$ with $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$ satisfying the specified limt.. **Note:** You could do all of these by choosing either x or x^2 for $f(x)$ and/or $g(x)$.

   (a) $(\infty - \infty = ?)$ $\lim_{x\to\infty} (f(x) - g(x)) = \infty$

   $f(x) = x^2 \qquad g(x) = x$

   (b) $(\infty - \infty = ?)$ $\lim_{x\to\infty} (f(x) - g(x)) = 0$

   $f(x) = x \qquad g(x) = x$

   (c) $(\infty - \infty = ?)$ $\lim_{x\to\infty} (f(x) - g(x)) = -\infty$

   $f(x) = x \qquad g(x) = x^2$

   (d) $(\frac{\infty}{\infty} = ?)$ $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

   $f(x) = x^2 \qquad g(x) = x$

   (e) $(\frac{\infty}{\infty} = ?)$ $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$

   $f(x) = x \qquad g(x) = x$

   (f) $(\frac{\infty}{\infty} = ?)$ $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

   $f(x) = x \qquad g(x) = x^2$