Math 1210 Lab 1 (Date: 8/24/2017)

Student Number:_____ Section:_____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed, unless stated otherwise.

1. (4 points) Find equations of the following lines. Write your answer in slope-intercept form $y = mx + b$.
   (a) The line with slope 4 that passes through the point $(1, -2)$.

   Answer:_____

   (b) The line passing through $(2, 3)$ and $(-1, 5)$.

   Answer:_____

Solution:

(a) We can use point-slope form to get $y + 2 = 4(x - 1)$ or $y = 4x - 6$.

(b) The slope of the line is $m = \frac{5-3}{-1-2} = -\frac{2}{3}$. Then we use point-slope form to get $y - 3 = -\frac{2}{3}(x - 2)$ or $y = -\frac{2}{3}x + \frac{13}{3}$.

2. (4 points) Are the following expressions equal for all values of x and y (for which both sides of the equality make sense)? Answer true (T) or false (F). If true, show algebraically. If false, give an example using specific values of x and y.

   (a) $(x+y)^2 = x^2 + y^2$

   Answer:_____

   (b) $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{x+y}$

   Answer:_____

Solution:

(a) False. Set $x = y = 1$. Then

$$(x+y)^2 = 2^2 = 4 \neq 2 = 1 + 1 = x^2 + y^2.$$

(b) False. Set $x = y = 1$. Then

$$\frac{1}{x} + \frac{1}{y} = 1 + 1 = 2 \neq \frac{1}{2} = \frac{1}{(1+1)} = \frac{1}{x+y}.$$

3. (14 points) Solve each expression by factoring the left-hand side. Answers should be real numbers only.

   (a) $3x^2 - 48 = 0$

   Answer:_____

   (b) $2x^2 - 4x - 6 = 0$

   Answer:_____

   (c) $x^2 + x - 3 = 0$

Answer:_____

(d) $x^3 - 7x^2 + 10x = 0$

Answer:_____

(e) $x^3 + 27 = 0$

Answer:_____

(f) $x^3 + x^2 - 4x - 4 = 0$

Answer:_____

Solution:

(a) $0 = 3(x^2 - 16) = 3(x+4)(x-4)$. So $x = \pm 4$.

(b) $0 = 2(x^2 - 2x - 3) = 2(x-3)(x+1)$. So $x = 3, -1$.

(c) Use the quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(1)(-3)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

(d) $0 = x(x^2 - 7x + 10) = x(x-5)(x-2)$. So $x = 0, 5, 2$.

(e) You can observe that $x = -3$ is a solution. Then we can do polynomial long division to find that

$$x^3 + 27 = (x+3)(x^2 - 3x + 9)$$

Since the discriminant of $x^2 - 3x + 9$ is $3^2 - 4(1)(9) = -27 < 0$, there are no other real factors. So $x = -3$ is the only solution.

(f) For this, you can guess integer factors by looking at the factors (with either sign) of the constant term. So you can plug in $x = \pm 1, \pm 2, \pm 4$ to see which ones yield zero. It turns out that

$$0 = x^3 + x^2 - 4x - 4 = (x+2)(x-2)(x+1),$$

and so $x = \pm 2, -1$.

4. (8 points) Evaluate the following expressions or state that they are undefined.

(a) $\left(\frac{1}{10}\right)^{-2}$

Answer:_____

(b) $8^{2/3}$

Answer:_____

(c) $(-8)^{2/3}$

Answer:_____

(d) $(-4)^{3/2}$

Answer:_____

Solution:

(a) $\left(\frac{1}{10}\right)^{-2} = (10)^2 = 100.$

(b) $8^{2/3} = (8^{1/3})^2 = (2)^2 = 4.$

(c) $(-8)^{2/3} = ((-8)^{1/3})^2 = (-2)^2 = 4.$

(d) Undefined. Since $(-4)^{3/2} = ((-4)^{1/2})^3$ or $(-4)^{3/2} = ((-4)^3)^{1/2} = (-64)^{1/2}$ and either way you can take the square root of a negative number.

5. (6 points) Evaluate the following, without a calculator.

(a) $\cos(\frac{2\pi}{3})$

Answer:_____

(b) $\cos(\frac{-\pi}{6})$

Answer:_____

(c) $\arcsin(\frac{-\sqrt{2}}{2})$

Answer:_____

Solution:

(a) $-\frac{1}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{7\pi}{4} = -\frac{\pi}{4}$

6. (6 points) Solve the following trigonometric equations, giving all solutions for θ in $0 \leq \theta < 2\pi$.

(a) $\sin^2\theta = \frac{1}{4}$

Answer:_____

(b) $\sin\theta\cos\theta = \cos\theta$

Answer:_____

Solution:

(a) If $\sin^2\theta = \frac{1}{4}$, then $\sin\theta = \pm\frac{1}{2}$. The equation $\sin\theta = \frac{1}{2}$ has solutions $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, while the equation $\sin\theta = -\frac{1}{2}$ has solutions $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$. So $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

(b) Since

$$0 = \sin\theta\cos\theta - \cos\theta = \cos\theta(\sin\theta - 1)$$

we will have solutions if $\cos\theta = 0$ (which happens at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$) or if $\sin\theta = 1$ (which happens at $\frac{\pi}{2}$). So the solutions are $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$.

7. (8 points) Find the domain of each function. Give your answer in interval notation.

(a) $f(x) = x + 3$

Answer:_____

(b) $f(x) = \dfrac{x^2 - 9}{x - 3}$

Answer:_____

(c) $f(x) = \sqrt{16 - x^3}$

Answer:_____

(d) $f(x) = \dfrac{1}{\sqrt{16 - x^3}}$

Answer:_____

Solution:

(a) $(-\infty, \infty)$

(b) $(-\infty, 3) \cup (3, \infty)$

(c) $(-\infty, \sqrt[3]{16}]$

(d) $(-\infty, \sqrt[3]{16})$

8. (15 points) Assume that $f(x) = x^2 + x - 1, g(x) = x + 2$, and h is a nonzero number. Simplify the following expressions.

(a) $f(g(x))$

Answer:_____

(b) $(g \circ f)(x)$

Answer:_____

(c) $(f \circ f)(x)$

Answer:_____

(d) $f(x + h)$

Answer:_____

(e) $\dfrac{f(x + h) - f(x)}{h}$

Answer:_____

Solution:

(a) $f(g(x)) = (x+2)^2 + (x+2) - 1 = x^2 + 5x + 5$

(b) $(g \circ f)(x) = g(f(x)) = (x^2 + x - 1) + 2 = x^2 + x + 1$

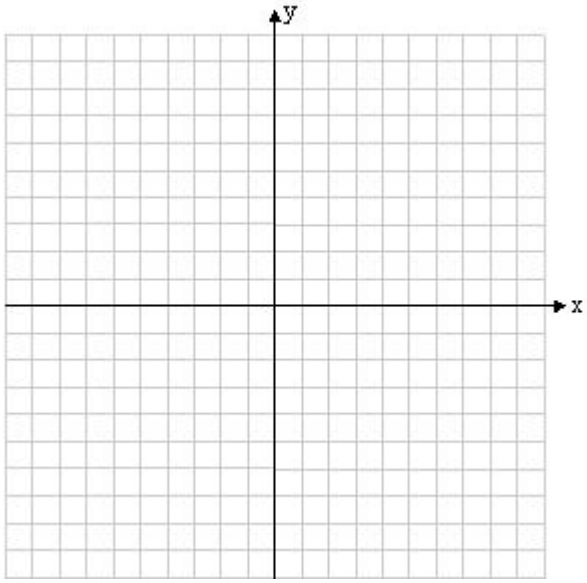
(c) $(f \circ f)(x) = f(f(x)) = (x^2 + x - 1)^2 + (x^2 + x - 1) - 1 = x^4 + 2x^3 - x - 1$

(d) $f(x+h) = (x+h)^2 + (x+h) - 1 = x^2 + 2hx + h^2 + x + h - 1 =$
$x^2 + (2h+1)x + h^2 + h - 1.$

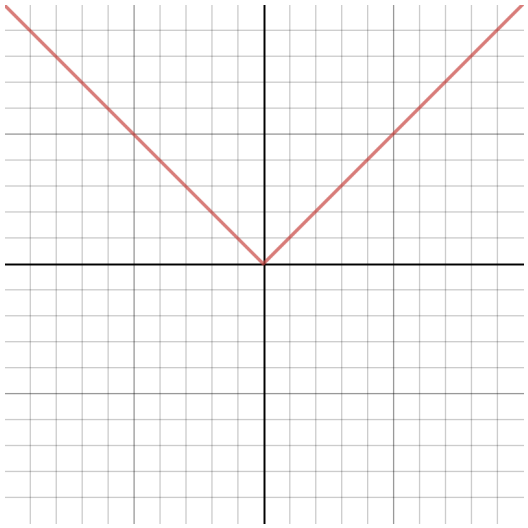
(e) $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+(x+h)-1-(x^2+x-1)}{h} = \frac{2xh+h^2+h}{h} = 2x + h + 1.$

9. (4 points) Sketch graph of the following piecewise-defined function. What is another way to write this function?

$$f(x) = \begin{cases} -x & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$



Solution: $f(x) = |x|$



10. (5 points) Explain why the Pythagorean Theorem implies $sec^2\theta - tan^2\theta = 1$.

Solution: $sin^2\theta + cos^2\theta = 1 \rightarrow \dfrac{sin^2\theta}{cos^2\theta} + \dfrac{cos^2\theta}{cos^2\theta} = \dfrac{1}{cos^2\theta}$

$tan^2\theta + 1 = sec^2\theta$

$sec^2\theta - tan^2\theta = 1$