1. (4 points) Find equations of the following lines. Write your answer in slope-intercept form \( y = mx + b \).

(a) The line with slope 4 that passes through the point \((1, -2)\).

Answer: 

(b) The line passing through \((2, 3)\) and \((-1, 5)\).

Answer:

Solution:
(a) We can use point-slope form to get \( y + 2 = 4(x - 1) \) or \( y = 4x - 6 \).

(b) The slope of the line is \( m = \frac{5 - 3}{1 - 2} = -\frac{2}{3} \). Then we use point-slope form to get \( y - 3 = -\frac{2}{3}(x - 2) \) or \( y = -\frac{2}{3}x + \frac{13}{3} \).
2. (4 points) Are the following expressions equal for all values of \( x \) and \( y \) (for which both sides of the equality make sense)? Answer true (T) or false (F). If true, show algebraically. If false, give an example using specific values of \( x \) and \( y \).

(a) \((x + y)^2 = x^2 + y^2\)

Answer: ________________

(b) \(\frac{1}{x} + \frac{1}{y} = \frac{1}{x + y}\)

Answer: ________________

Solution:
(a) False. Set \( x = y = 1 \). Then

\[
(x + y)^2 = 2^2 = 4 \neq 2 = 1 + 1 = x^2 + y^2.
\]

(b) False. Set \( x = y = 1 \). Then

\[
\frac{1}{x} + \frac{1}{y} = 1 + 1 = 2 \neq \frac{1}{2} = \frac{1}{(1 + 1)} = \frac{1}{x + y}.
\]

3. (14 points) Solve each expression by factoring the left-hand side. Answers should be real numbers only.

(a) \(3x^2 - 48 = 0\)

Answer: ________________

(b) \(2x^2 - 4x - 6 = 0\)

Answer: ________________

(c) \(x^2 + x - 3 = 0\)

Answer: ________________
(d) \( x^3 - 7x^2 + 10x = 0 \)

Answer: 

(e) \( x^3 + 27 = 0 \)

Answer: 

(f) \( x^3 + x^2 - 4x - 4 = 0 \)

Answer: 

Solution:

(a) \( 0 = 3(x^2 - 16) = 3(x + 4)(x - 4) \). So \( x = \pm 4 \).

(b) \( 0 = 2(x^2 - 2x - 3) = 2(x - 3)(x + 1) \). So \( x = 3, -1 \).

(c) Use the quadratic formula:

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(1)(-3)}}{2} = \frac{-1 \pm \sqrt{13}}{2}
\]

(d) \( 0 = x(x^2 - 7x + 10) = x(x - 5)(x - 2) \). So \( x = 0, 5, 2 \).

(e) You can observe that \( x = -3 \) is a solution. Then we can do polynomial long division to find that

\[ x^3 + 27 = (x + 3)(x^2 - 3x + 9) \]

Since the discriminant of \( x^2 - 3x + 9 \) is \( 3^2 - 4(1)(9) = -27 < 0 \), there are no other real factors. So \( x = -3 \) is the only solution.
(f) For this, you can guess integer factors by looking at the factors (with either
sign) of the constant term. So you can plug in \( x = \pm 1, \pm 2, \pm 4 \) to see which
ones yield zero. It turns out that

\[
0 = x^3 + x^2 - 4x - 4 = (x + 2)(x - 2)(x + 1),
\]

and so \( x = \pm 2, -1 \).

4. (8 points) Evaluate the following expressions or state that they are undefined.

(a) \( \left( \frac{1}{10} \right)^{-2} \)

Answer: ______________________

(b) \( 8^{2/3} \)

Answer: ______________________

(c) \( (-8)^{2/3} \)

Answer: ______________________

(d) \( (-4)^{3/2} \)

Answer: ______________________

Solution:

(a) \( \left( \frac{1}{10} \right)^{-2} = (10)^2 = 100. \)

(b) \( 8^{2/3} = (8^{1/3})^2 = (2)^2 = 4. \)

(c) \( (-8)^{2/3} = ((-8)^{1/3})^2 = (-2)^2 = 4. \)

(d) Undefined. Since \( (-4)^{3/2} = ((-4)^{1/2})^3 \) or \( (-4)^{3/2} = ((-4)^3)^{1/2} = (-64)^{1/2} \) and
either way you can take the square root of a negative number.
5. (6 points) Evaluate the following, without a calculator.

(a) \( \cos\left(\frac{2\pi}{3}\right) \)

Answer: \( \frac{-1}{2} \)

(b) \( \cos\left(\frac{-\pi}{6}\right) \)

Answer: \( \frac{\sqrt{3}}{2} \)

(c) \( \arcsin\left(\frac{-\sqrt{2}}{2}\right) \)

Answer: \( \frac{7\pi}{4} = -\frac{\pi}{4} \)

Solution:

(a) \( -\frac{1}{2} \)

(b) \( \frac{\sqrt{3}}{2} \)

(c) \( \frac{7\pi}{4} = -\frac{\pi}{4} \)
6. (6 points) Solve the following trigonometric equations, giving all solutions for \( \theta \) in \( 0 \leq \theta < 2\pi \).

(a) \( \sin^2 \theta = \frac{1}{4} \)

Answer:

(b) \( \sin \theta \cos \theta = \cos \theta \)

Answer:

**Solution:**

(a) If \( \sin^2 \theta = \frac{1}{4} \), then \( \sin \theta = \pm \frac{1}{2} \). The equation \( \sin \theta = \frac{1}{2} \) has solutions \( \theta = \frac{\pi}{6}, \frac{5\pi}{6} \). While the equation \( \sin \theta = -\frac{1}{2} \) has solutions \( \theta = \frac{7\pi}{6}, \frac{11\pi}{6} \). So \( \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \).

(b) Since

\[
0 = \sin \theta \cos \theta - \cos \theta = \cos \theta (\sin \theta - 1)
\]

we will have solutions if \( \cos \theta = 0 \) (which happens at \( \frac{\pi}{2} \) and \( \frac{3\pi}{2} \)) or if \( \sin \theta = 1 \) (which happens at \( \frac{\pi}{2} \)). So the solutions are \( \theta = \frac{\pi}{2}, \frac{3\pi}{2} \).
7. (8 points) Find the domain of each function. Give your answer in interval notation.

(a) $f(x) = x + 3$

(b) $f(x) = \frac{x^2 - 9}{x - 3}$

(c) $f(x) = \sqrt{16 - x^3}$

(d) $f(x) = \frac{1}{\sqrt{16 - x^3}}$

Solution:

(a) $(-\infty, \infty)$

(b) $(-\infty, 3) \cup (3, \infty)$

(c) $(-\infty, \sqrt{16}]$

(d) $(-\infty, \sqrt[3]{16})$
8. (15 points) Assume that \( f(x) = x^2 + x - 1, g(x) = x + 2, \) and \( h \) is a nonzero number. Simplify the following expressions.

(a) \( f(g(x)) \)

Answer: __________________________

(b) \( (g \circ f)(x) \)

Answer: __________________________

(c) \( (f \circ f)(x) \)

Answer: __________________________

(d) \( f(x + h) \)

Answer: __________________________

(e) \( \frac{f(x + h) - f(x)}{h} \)

Answer: __________________________
Solution:
(a) \( f(g(x)) = (x + 2)^2 + (x + 2) - 1 = x^2 + 5x + 5 \)
(b) \( (g \circ f)(x) = g(f(x)) = (x^2 + x - 1) + 2 = x^2 + x + 1 \)
(c) \( (f \circ f)(x) = f(f(x)) = (x^2 + x - 1)^2 + (x^2 + x - 1) - 1 = x^4 + 2x^3 - x - 1 \)
(d) \( f(x + h) = (x + h)^2 + (x + h) - 1 = x^2 + 2hx + h^2 + x + h - 1 = \)
\hspace{1cm} \( x^2 + (2h + 1)x + h^2 + h - 1 \).
(e) \( \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + (x+h) - 1 - (x^2 + x - 1)}{h} = \frac{2hx + h^2 + h}{h} = 2x + h + 1. \)
9. (4 points) Sketch graph of the following piecewise-defined function. What is another way to write this function?

\[ f(x) = \begin{cases} 
-x & \text{if } x < 0 \\
 x & \text{if } x \geq 0 
\end{cases} \]

\textbf{Solution: } f(x) = |x|
10. (5 points) Explain why the Pythagorean Theorem implies \( \sec^2 \theta - \tan^2 \theta = 1 \).

Solution: 
\[
\sin^2 \theta + \cos^2 \theta = 1 \rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}
\]

\[\tan^2 \theta + 1 = \sec^2 \theta\]

\[\sec^2 \theta - \tan^2 \theta = 1\]