MATH 1210 FALL 2017, HOMEWORK 5

The graded problems were 2.3.58, 2.4.26, 2.5.52.

$\boxed{2.3.58}$ At $x = a$, the equation of the tangent line to $y = x^2$ is given by $y - a^2 = 2a(x - a)$. The problem translates to finding a such that this line contains the point $(4, 15)$, i.e. such that

$$15 - a^2 = 2a(4 - a) = 8a - a^2.$$

Rearranging, we get $0 = a^2 - 8a + 15 = (a - 3)(a - 5)$, so a is 3 or 5. But the problem specifies that the space-traveler is moving left to right, so we must have $\boxed{a = 3}$.

$\boxed{2.4.26}$ The derivative of $y = \tan^2(x)$ is $y' = 2\tan(x)\sec^2(x)$. Since $\sec(x)$ is never zero, this is only zero when $\sin(x) = 0$, i.e. when $\boxed{x \text{ is an integer multiple of } \pi}$.

$\boxed{2.5.52}$ $F(y)$ is some unspecified differentiable function of y, and we are asked to compute (using the chain rule)

$$\frac{d}{dy}\left(y^2 + \frac{1}{F(y^2)}\right) = \boxed{2y + \frac{-1}{F(y^2)^2} \cdot F'(y^2) \cdot 2y}.$$

$\boxed{\text{Quiz}}$ Let $f(x) = \sqrt{\sin(x)}$.

(a) since the domain of $\sqrt{x}$ is all $x \geq 0$, the domain of $f(x)$ will be those x such that $\sin(x) \geq 0$, i.e. the $\boxed{\text{union of the closed intervals of the form } [2n\pi, (2n+1)\pi], \text{ for all integers } n}$. (eg, $[0, \pi]$ and $[2\pi, 3\pi]$ are allowed, but not $[\pi, 2\pi]$.)

(b)

$$\boxed{f'(x) = \frac{\cos(x)}{2\sqrt{\sin(x)}},}$$

valid in the $\boxed{\textit{open} \text{ intervals } (2n\pi, (2n+1)\pi), \text{ for all integers } n}$ (not at the endpoints).

(c) The equation of the tangent line is $y - f(\pi/6) = f'(\pi/6)(x - \pi/6)$. Since $\sin(\pi/6) = 1/2$ and $\cos(\pi/6) = \sqrt{3}/2$, we find

$$\boxed{y - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}}(x - \frac{\pi}{6})}.$$

(d) $\boxed{\text{Yes.}}$ The formula computed in (b) shows that $\lim_{x\to 0^+} f'(x) = \infty$, so there is some value of x_0, which we can clearly assume to be in $(0, \frac{\pi}{2})$, such that $f'(x_0) > 1000$. Since the function $f'(x)$ is continuous on $[x_0, \frac{\pi}{2}]$, and 1000 lies between $f'(x_0)$ and $f'(\frac{\pi}{2}) = 0$, the intermediate value theorem ensures that for some $c \in (x_0, \frac{\pi}{2})$, we must have $f'(c) = 1000$.