The graded problems were 2.3.58, 2.4.26, 2.5.52.

2.3.58 At \( x = a \), the equation of the tangent line to \( y = x^2 \) is given by \( y - a^2 = 2a(x - a) \). The problem translates to finding \( a \) such that this line contains the point (4, 15), i.e. such that

\[
15 - a^2 = 2a(4 - a) = 8a - a^2.
\]

Rearranging, we get

\[
0 = a^2 - 8a + 15 = (a - 3)(a - 5),
\]

so \( a \) is 3 or 5. But the problem specifies that the space-traveler is moving left to right, so we must have \( a = 3 \).

2.4.26 The derivative of \( y = \tan^2(x) \) is

\[
y' = 2 \tan(x) \sec^2(x).
\]

Since \( \sec(x) \) is never zero, this is only zero when \( \sin(x) = 0 \), i.e. when \( x \) is an integer multiple of \( \pi \).

2.5.52 \( F(y) \) is some unspecified differentiable function of \( y \), and we are asked to compute (using the chain rule)

\[
\frac{d}{dy} \left( y^2 + \frac{1}{F(y^2)} \right) = 2y + \frac{-1}{F(y^2)} \cdot F'(y^2) \cdot 2y.
\]

Quiz Let \( f(x) = \sqrt{\sin(x)} \).

(a) since the domain of \( \sqrt{x} \) is all \( x \geq 0 \), the domain of \( f(x) \) will be those \( x \) such that

\[
\sin(x) \geq 0,
\]

i.e. the union of the closed intervals of the form \([2n\pi, (2n + 1)\pi]\), for all integers \( n \).

(eg, \([0, \pi]\) and \([2\pi, 3\pi]\) are allowed, but not \([\pi, 2\pi]\).)

(b) \[
f'(x) = \frac{\cos(x)}{2 \sqrt{\sin(x)}}
\]

valid in the open intervals \((2n\pi, (2n + 1)\pi]\), for all integers \( n \) (not at the endpoints).

(c) The equation of the tangent line is

\[
y - f'(\pi/6) = f'(\pi/6)(x - \pi/6).
\]

Since \( \sin(\pi/6) = 1/2 \) and \( \cos(\pi/6) = \sqrt{3}/2 \), we find

\[
y - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2 \sqrt{2}} \left( x - \frac{\pi}{6} \right).
\]

(d) Yes. The formula computed in (b) shows that \( \lim_{x \to 0^+} f'(x) = \infty \), so there is some value of \( x_0 \), which we can clearly assume to be in \((0, \pi/2]\), such that \( f'(x_0) > 1000 \). Since the function \( f'(x) \) is continuous on \([x_0, \pi/2]\), and 1000 lies between \( f'(x_0) \) and \( f'(\pi/2) = 0 \), the intermediate value theorem ensures that for some \( c \in (x_0, \pi/2) \), we must have \( f'(c) = 1000 \).