MATH 1210 FALL 2017, HOMEWORK 4

The graded problems were 2.1.16, 2.2.22, 2.2.30

$\boxed{2.1.16}$ $s(t) = -t^2 + 4t$ gives the particle's position as a function of time. Then $s'(t) = -2t + 4$, and we are asked when its velocity is 0, i.e. when $0 = -2t + 4$. This occurs at $\boxed{t = 2 \text{ seconds}}$.

$\boxed{2.2.22}$

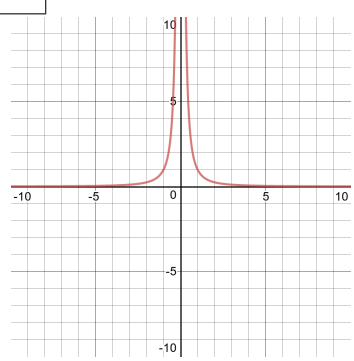
$$\lim_{h\to 0} \frac{H(x+h) - H(x)}{h} = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+4} - \sqrt{x^2+4}}{h} \cdot \frac{\sqrt{(x+h)^2+4} + \sqrt{x^2+4}}{\sqrt{(x+h)^2+4} + \sqrt{x^2+4}}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 + 4 - (x^2+4)}{h \cdot (\sqrt{(x+h)^2+4} + \sqrt{x^2+4})} = \lim_{h\to 0} \frac{2xh + h^2}{h \cdot (\sqrt{(x+h)^2+4} + \sqrt{x^2+4})}$$

$$= \lim_{h\to 0} \frac{2x+h}{\sqrt{(x+h)^2+4} + \sqrt{x^2+4}} = \boxed{\frac{x}{\sqrt{x^2+4}}}$$

$\boxed{2.2.30}$ This is the derivative at $\boxed{x = 3}$ of the function $\boxed{f(x) = x^3 + x}$, or indeed any function of the form $\boxed{f(x) = x^3 + x + c}$ where c is some constant.

$\boxed{\text{Quiz}}$ (a) Graph of $f(x) = \frac{1}{x^2}$:



(b) $f'(x)$, calculated *from the limit definition of the derivative*:

$$\lim_{y\to x} \frac{f(y) - f(x)}{y - x} = \lim_{y\to x} \frac{1/y^2 - 1/x^2}{y - x} = \lim_{y\to x} \frac{(x-y)(x+y)}{x^2y^2(y-x)} = \frac{-2x}{x^4} = \boxed{\frac{-2}{x^3}}$$

(Using the $\lim_{h\to}$ version of the limit definition is of course fine too.)

(c) Here are a few things one might say (full credit for saying at least two correct things):

- The function has positive slope for $x < 0$ and negative slope for $x > 0$.
- As $x \to \pm\infty$, the slope approaches 0.
- As $x \to 0^+$, the slope approaches $-\infty$. As $x \to 0^-$, the slope approaches $+\infty$.