The graded problems were 2.1.16, 2.2.22, 2.2.30

2.1.16 \( s(t) = -t^2 + 4t \) gives the particle’s position as a function of time. Then \( s'(t) = -2t + 4 \), and we are asked when its velocity is 0, i.e. when \( 0 = -2t + 4 \). This occurs at \( t = 2 \) seconds.

2.2.22 \[
\lim_{h \to 0} \frac{H(x + h) - H(x)}{h} = \lim_{h \to 0} \frac{\sqrt{(x + h)^2 + 4} - \sqrt{x^2 + 4}}{h} \cdot \frac{\sqrt{(x + h)^2 + 4} + \sqrt{x^2 + 4}}{\sqrt{(x + h)^2 + 4} + \sqrt{x^2 + 4}}
\]
\[
= \lim_{h \to 0} \frac{(x + h)^2 + 4 - (x^2 + 4)}{h \cdot (\sqrt{(x + h)^2 + 4} + \sqrt{x^2 + 4})} = \lim_{h \to 0} \frac{2xh + h^2}{h \cdot (\sqrt{(x + h)^2 + 4} + \sqrt{x^2 + 4})}
\]
\[
= \lim_{h \to 0} \frac{2x + h}{\sqrt{(x + h)^2 + 4} + \sqrt{x^2 + 4}} = \frac{x}{\sqrt{x^2 + 4}}
\]

2.2.30 This is the derivative at \( x = 3 \) of the function \( f(x) = x^3 + x + c \), or indeed any function of the form \( f(x) = x^3 + x + c \) where \( c \) is some constant.

Quiz

(a) Graph of \( f(x) = \frac{1}{x} \):

(b) \( f'(x) \), calculated from the limit definition of the derivative:
\[
\lim_{y \to x} \frac{f(y) - f(x)}{y - x} = \lim_{y \to x} \frac{1/y^2 - 1/x^2}{y - x} = \lim_{y \to x} \frac{(x - y)(x + y)}{x^2y^2(y - x)} = -\frac{2x}{x^4} = -\frac{2}{x^3}
\]

(Using the \( \lim_{h \to 0} \) version of the limit definition is of course fine too.)

(c) Here are a few things one might say (full credit for saying at least two correct things):

- The function has positive slope for \( x < 0 \) and negative slope for \( x > 0 \).
- As \( x \to \pm \infty \), the slope approaches 0.
- As \( x \to 0^+ \), the slope approaches \( -\infty \). As \( x \to 0^- \), the slope approaches \( +\infty \).