Thus, while $\lim_{x\to 2}[\![x]\!]$ does not exist, it is correct to write (look at the graph in Figure 7)

$$\lim_{x\to 2^-}[\![x]\!] = 1 \quad \text{and} \quad \lim_{x\to 2^+}[\![x]\!] = 2$$

We believe that you will find the following theorem quite reasonable.

Theorem A

$\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c^-} f(x) = L$ and $\lim_{x\to c^+} f(x) = L$.

Figure 10 should give additional insight. Two of the limits do not exist, although all but one of the one-sided limits exist.

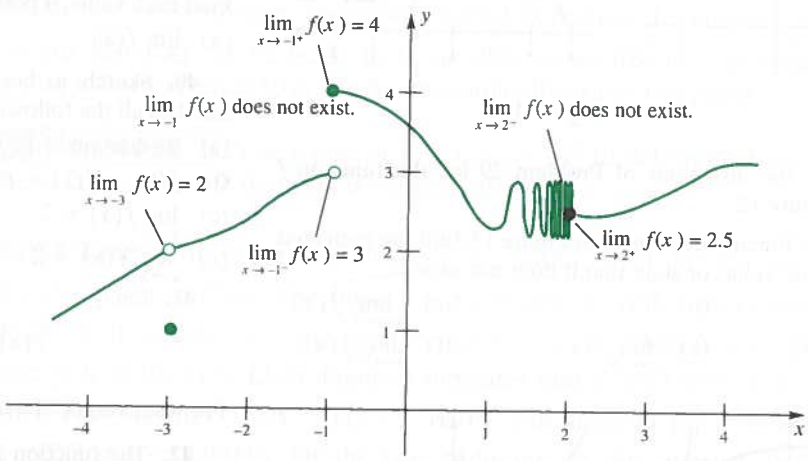

Figure 10

Concepts Review

1. $\lim_{x\to c} f(x) = L$ means that $f(x)$ gets close to _____ when x gets sufficiently close to (but is different from) _____.
2. Let $f(x) = (x^2 - 9)/(x - 3)$ and note that $f(3)$ is undefined. Nevertheless, $\lim_{x\to 3} f(x) =$ _____.
3. $\lim_{x\to c^+} f(x) = L$ means that $f(x)$ gets near to _____ when x approaches c from the _____.
4. If both $\lim_{x\to c^-} f(x) = M$ and $\lim_{x\to c^+} f(x) = M$, then _____.

Problem Set 1.1

In Problems 1–6, find the indicated limit.

1. $\lim_{x\to 3}(x - 5)$
2. $\lim_{t\to -1}(1 - 2t)$
3. $\lim_{x\to -2}(x^2 + 2x - 1)$
4. $\lim_{x\to -2}(x^2 + 2t - 1)$
5. $\lim_{t\to -1}(t^2 - 1)$
6. $\lim_{t\to -1}(t^2 - x^2)$

In Problems 7–18, find the indicated limit. In most cases, it will be wise to do some algebra first (see Example 2).

7. $\lim_{x\to 2}\dfrac{x^2 - 4}{x - 2}$
8. $\lim_{t\to -7}\dfrac{t^2 + 4t - 21}{t + 7}$
9. $\lim_{x\to -1}\dfrac{x^3 - 4x^2 + x + 6}{x + 1}$
10. $\lim_{x\to 0}\dfrac{x^4 + 2x^3 - x^2}{x^2}$
11. $\lim_{x\to -t}\dfrac{x^2 - t^2}{x + t}$
12. $\lim_{x\to 3}\dfrac{x^2 - 9}{x - 3}$
13. $\lim_{t\to 2}\dfrac{\sqrt{(t + 4)(t - 2)^4}}{(3t - 6)^2}$
14. $\lim_{t\to 7^+}\dfrac{\sqrt{(t - 7)^3}}{t - 7}$
15. $\lim_{x\to 3}\dfrac{x^4 - 18x^2 + 81}{(x - 3)^2}$
16. $\lim_{u\to 1}\dfrac{(3u + 4)(2u - 2)^3}{(u - 1)^2}$
17. $\lim_{h\to 0}\dfrac{(2 + h)^2 - 4}{h}$
18. $\lim_{h\to 0}\dfrac{(x + h)^2 - x^2}{h}$

GC *In Problems 19–28, use a calculator to find the indicated limit. Use a graphing calculator to plot the function near the limit point.*

19. $\lim_{x\to 0}\dfrac{\sin x}{2x}$
20. $\lim_{t\to 0}\dfrac{1 - \cos t}{2t}$
21. $\lim_{x\to 0}\dfrac{(x - \sin x)^2}{x^2}$
22. $\lim_{x\to 0}\dfrac{(1 - \cos x)^2}{x^2}$
23. $\lim_{t\to 1}\dfrac{t^2 - 1}{\sin(t - 1)}$
24. $\lim_{x\to 3}\dfrac{x - \sin(x - 3) - 3}{x - 3}$
25. $\lim_{x\to \pi}\dfrac{1 + \sin(x - 3\pi/2)}{x - \pi}$
26. $\lim_{t\to 0}\dfrac{1 - \cot t}{1/t}$
27. $\lim_{x\to \pi/4}\dfrac{(x - \pi/4)^2}{(\tan x - 1)^2}$
28. $\lim_{u\to \pi/2}\dfrac{2 - 2\sin u}{3u}$