As a set, the rotation group SO(3) is homeomorphic to the 3-dimensional Real Projective space, $\mathbb{R}\mathrm{P}^3$, the first fundamental group of which is $\mathbb{Z}_2$. This means that the square of any loop is homotopic to a point (or shrinkable). This topological property of the rotation group has a very interesting physical presentation.

For a Persian article about this see the following
`http:/www.iasbs.ac.ir/faculty/shahram/rotentg.ps.gz`

A Shariati

The following text is from the famous MTW book *Gravitation*.

> Paint each face of a cube a different color, then connect each corner of the cube to the corresponding corner of the room with an elastic thread (Fig). Now rotate the cube through $2\pi = 360°$ degree. The threads become tangled. Nothing one can do will untangle them. It is impossible for every thread to proceed on its way in a straight line. Now rotate the cube about the same axix by a furthur 2π. The threads become still more tangled. However, a little work now completely straightens out the tangle (fig). Every thread runs as it did in the beginning in a straight line from its corner of the cube to the corresponding corner of the room. More generally, rotations by $\pm 4\pi, \pm 8\pi, \ldots$, leave the cube in its standard "orientation-entanglement relation" with its surroundings, whereas rotations by $\pm 2\pi, \pm 6\pi, \pm 10\pi, \ldots$, restore to the cube only its orientation, not its orientation-entanglement with its surrondings. Evidently there is something about the geometry of orientation that is not fully taken into account in the usual concept of orientation; hence the concept of "orientation-entanglement relation" or (briefer term!) "version" (Latin *versor*, turn). Whether there is also a detectable difference in physics (contact potential between a metallic object ant its metallic surroundings, for example) for two inequivalent versions of an object is not known [Aharanov and Susskind (1967)].
> Gravitation; *C.W. Misner, K.S. Thorne, J.A. Wheeler*; p. 1147 (W.H. Freeman and Company, San Francisco, 1973)

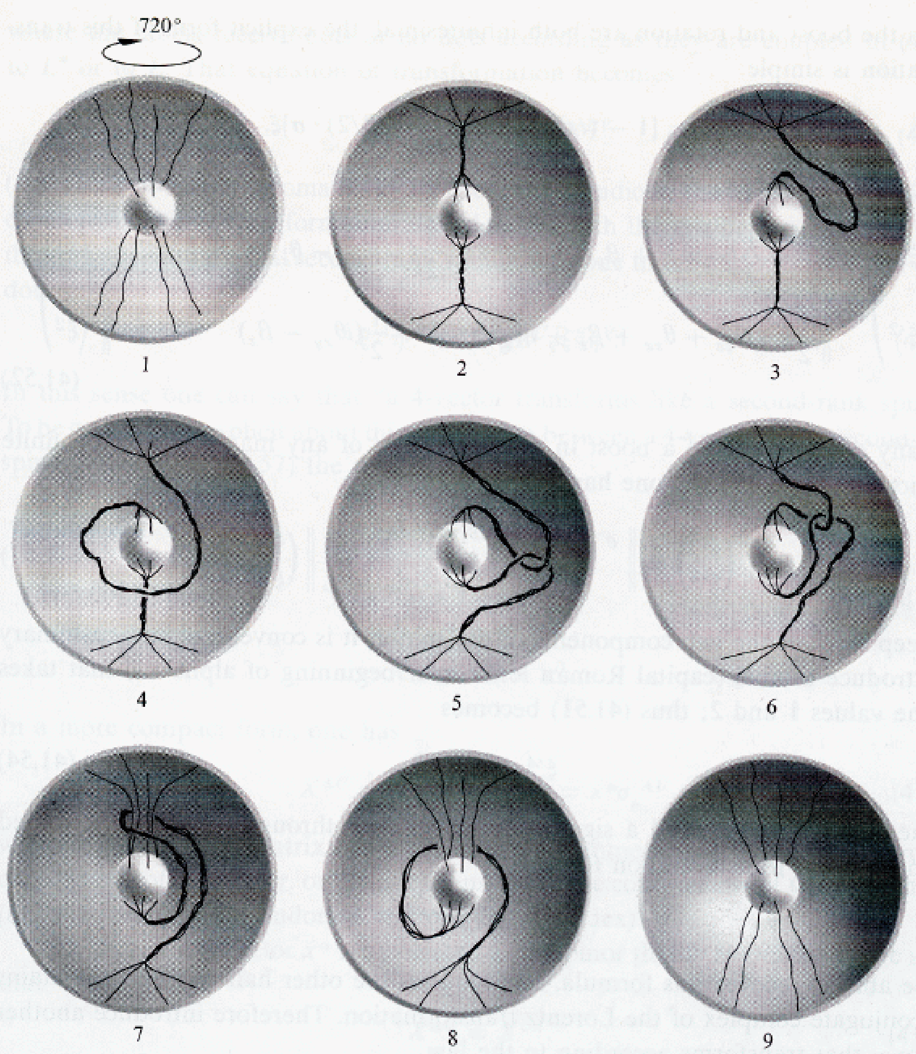

Figure 41.6.
An object is connected to its surroundings by elastic threads as in Figure 41.5. (Eight are shown here; any number could be used.) Rotating the object through 720° and then following the procedure outlined (Edward McDonald) in frames 2–8 (with the object remaining fixed), one finds that the connecting threads are left disentangled, as in frame 9 (lower right).