Math 2210-1

Notes of 01/23/24

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11.6 Lines and Tangents

- The simplest curve is a line.
- A line is determined by a fixed point

$$P_0 = (x_0, y_0, z_0)$$

  and a fixed vector

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

  contained in the line.

- A line though the point P in the direction $\mathbf{v}$ can be written as

$$\mathbf{r} = \vec{OP_0} + t\mathbf{v} = \mathbf{r}_0 + t\mathbf{v}$$

  where $\mathbf{r}_0 = \vec{OP_0}$.

- This is a **parametric** representation of the line. In component form it is given by

$$x = x_0 + at, \qquad y = y_0 + bt, \qquad z = z_0 + ct.$$

- Example 1. Write an equation for the line through the points $(3, -2, 4)$ and $(5, 6, -2)$.

- Modify Example 1 such that t gives the distance from the point $(3, -2, 4)$.

Symmetric Equations of a Line

- In the previous example we got the parametric equations

$$x = 3+2t, \qquad y = -2+8t, \quad \text{and} \quad z = 4-6t.$$

- Suppose (x, y, z) is a point on that line. Then for some value of t we get the values x, y, z, and t is the same for all three values.
- We can solve each equation for t:

$$\begin{array}{ccccccc} x & = & 3+2t & \Longrightarrow & t & = & \frac{x-3}{2} \\ y & = & -2+8t & \Longrightarrow & t & = & \frac{y+2}{8} \\ z & = & 4-6t & \Longrightarrow & t & = & \frac{z-4}{-6} \end{array}$$

- Eliminating t gives the **symmetric equations of the line:**

$$\frac{x-3}{2} = \frac{y+2}{8} = \frac{z-4}{-6}.$$

- In general, the symmetric equations of the line

$$\mathbf{r}(t) =< x_0, y_0, z_0 > +t < a, b, c >$$

are

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}.$$

- Example 3, textbook. Find the symmetric equations of the line where the two planes

$$2x - y - 5z = -14$$

and

$$4x + 5y + 4z = 28$$

intersect.

- Once you have a line and a point you can find the plane that is perpendicular to the line at that point.
- Example: Find an equation for the tangent of the curve, and the plane that is perpendicular to that tangent,

$$\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$$

  at the point where $t = 2$.