Math 2210-1

Notes of 8/24/18

Announcements

- You should have received a message from me with login info for WeBWorK.
- Email me, pa@math.utah.edu, if you are signed up for the class, and you have not received a message.
- ww email is sent through your Umail account.
- You can change your email address within ww.
- The first home work is open until Friday next week.
- hw 2 will open on Monday.
- You should finish each home work before the next one opens, so **you should finish hw 1 by Sunday night.**
- This morning: 100 people signed up for class, 5 people done completely, 51 done partially, 46 have not started answering question (but may of course have solved some or all problems).
- Today is the last day to register for the class without having to get permission.

- next Friday is the last day do drop the class without having to pay tuition.
- I'm in a rush after class since I have to go to another but you should have no trouble getting a hold of me at other times.

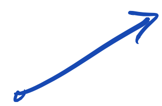
11.2 Vectors

- first major new concept this semester
- A **vector** has length (or magnitude) and direction.
- Examples include:
  - displacement
  - velocity
  - acceleration
  - force
- By contrast, a number has no direction. To emphasize the contrast with a vector, in this context a number is referred to as a **scalar**.
- Examples of scalars include:
  - temperature
  - pressure
  - density
  - speed

- Following the textbook, unless indicated otherwise, we use lower case bold face letters to denote vectors and ordinary lower case letters to denote scalars.

- You can think of a vector as an **arrow**. It has a **tail** and a **head** (or **tip**).

- We **add** two vectors **u** and **v** by attaching the tail of **v** to the head of **u**.



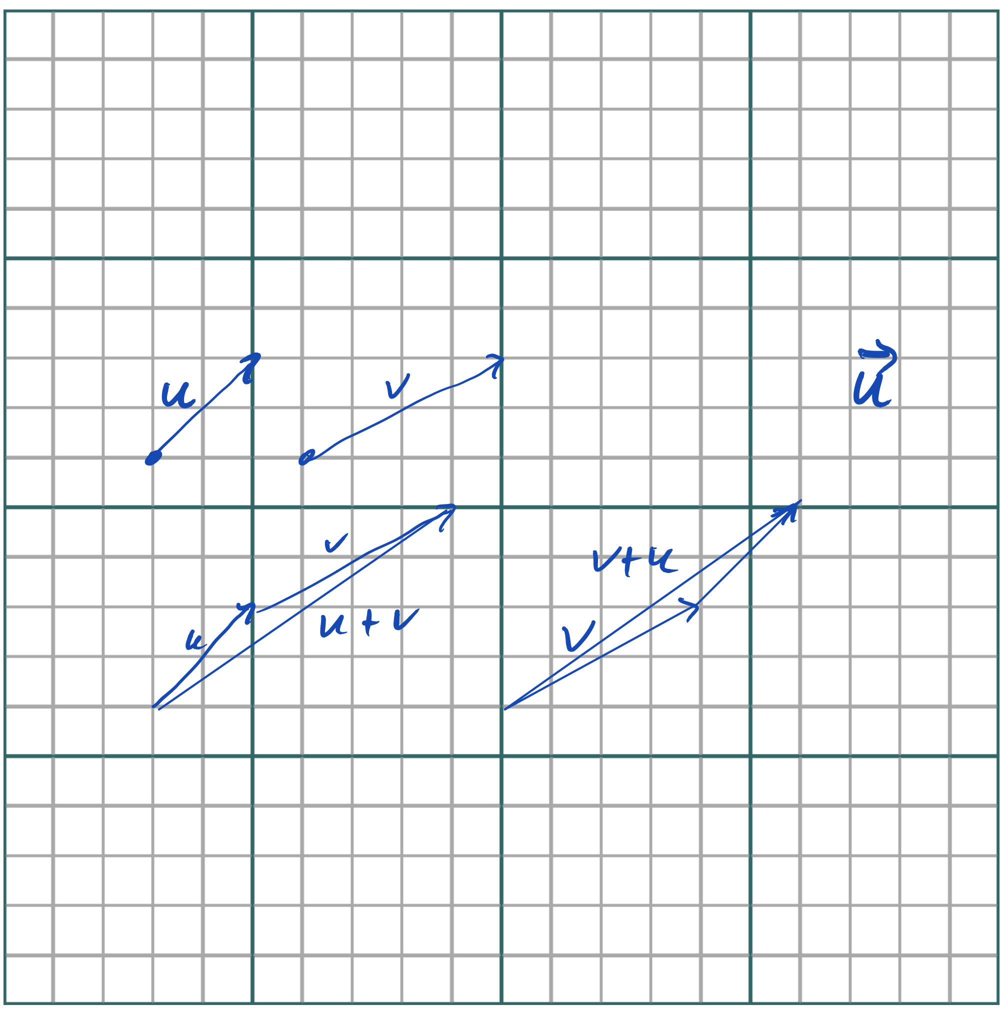


Figure 1. Vector Addition.

- The **Parallelogram Law** says that for all vectors $\mathbf{u}$ and $\mathbf{v}$:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

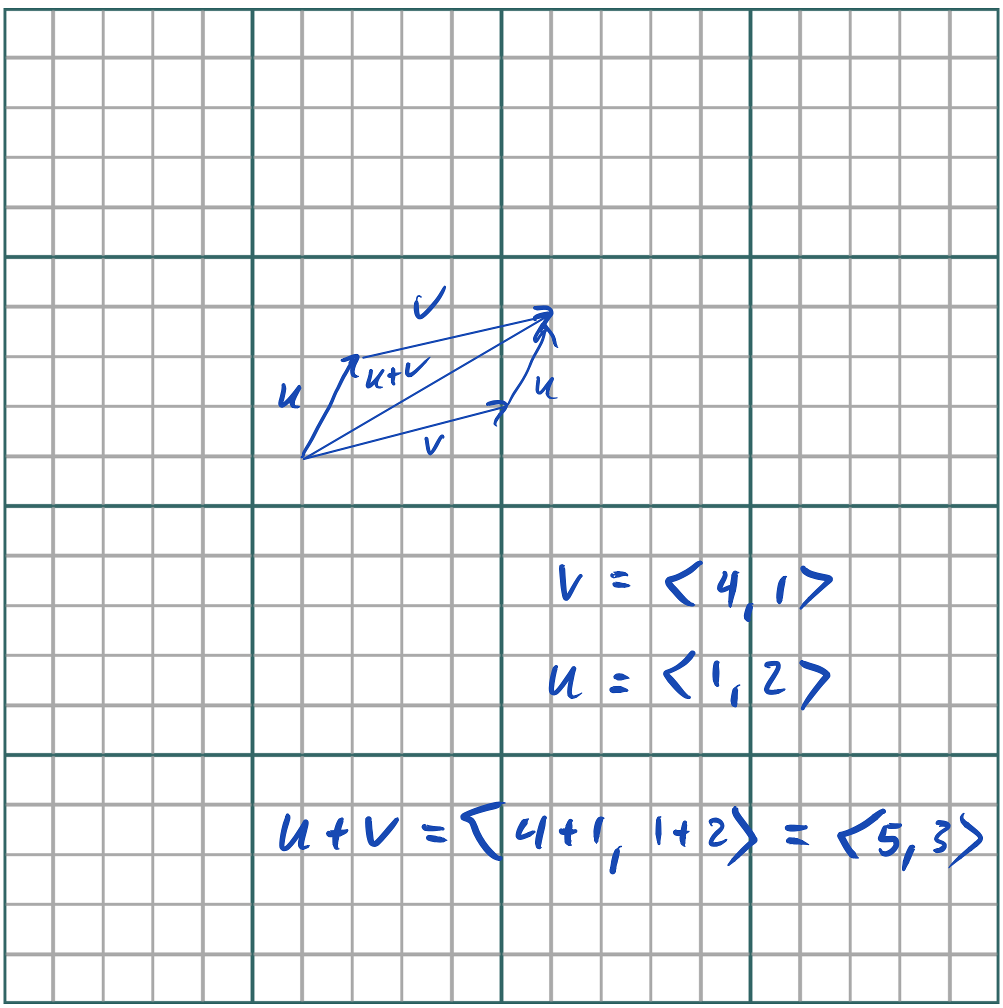


Figure 2. Parallelogram Law.

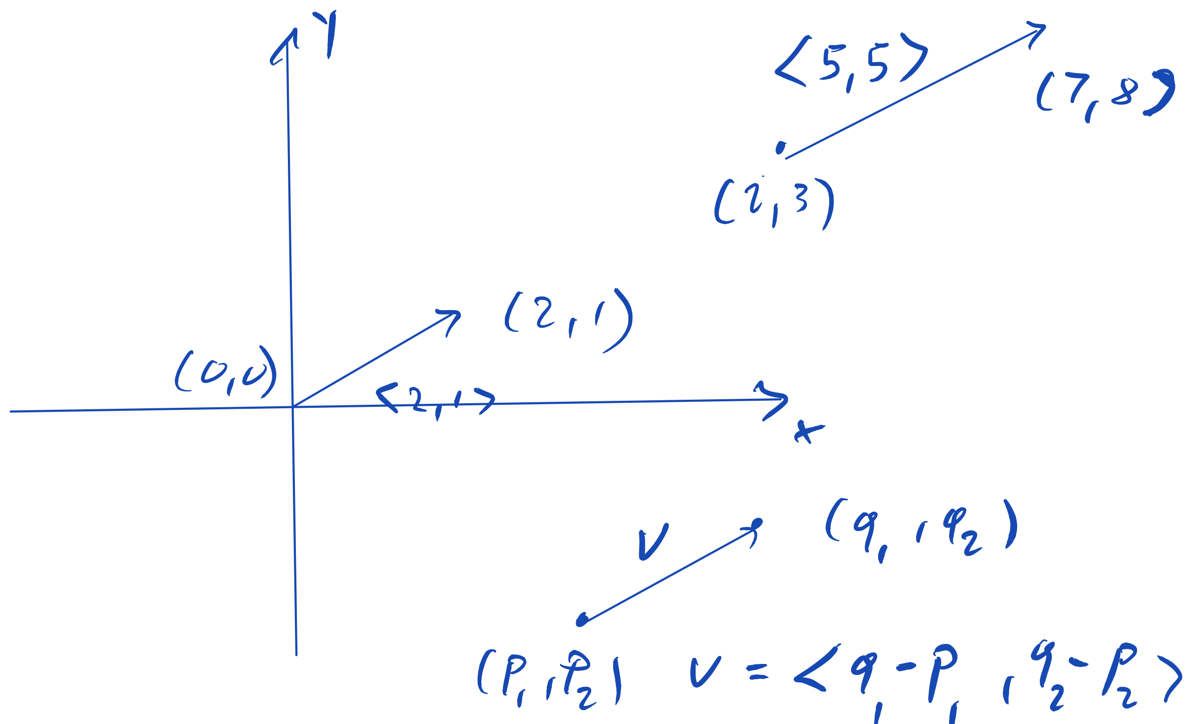
- We also write vectors as

$$\mathbf{v} = < v_1, v_2 >$$

in the plane or

$$\mathbf{v} =< v_1, v_2, v_3 >$$

in 3-space.



- A point (x, y) is often **identified** with the vector $< v_1, v_2 >$. This amounts to assuming the tail of the vector is the origin.

- A notation like $< v_1, v_2 >$ can also mean the set of all vectors with tail (p_1, p_2) and head (q_1, q_2) where

$$v_1 = q_1 - p_1 \quad \text{and} \quad v_2 = q_2 - p_2.$$

- This may appear very confusing, but it will be always clear from the context what is meant.

- Vectors can be added, and multiplied with scalars:

  $$\mathbf{v}+\mathbf{w} = < v_1, v_2 > + < w_1, w_2 > = < v_1+w_1, v_2+w_2 >$$

  and

  $$s\mathbf{v} = s < v_1, v_2 > = < sv_1, sv_2 >$$ $3\langle 1,2\rangle = \langle 3,6\rangle$

- Similarly for vectors in $\mathbb{R}^3$. (We'll do this a lot: state a general property only for vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$.)

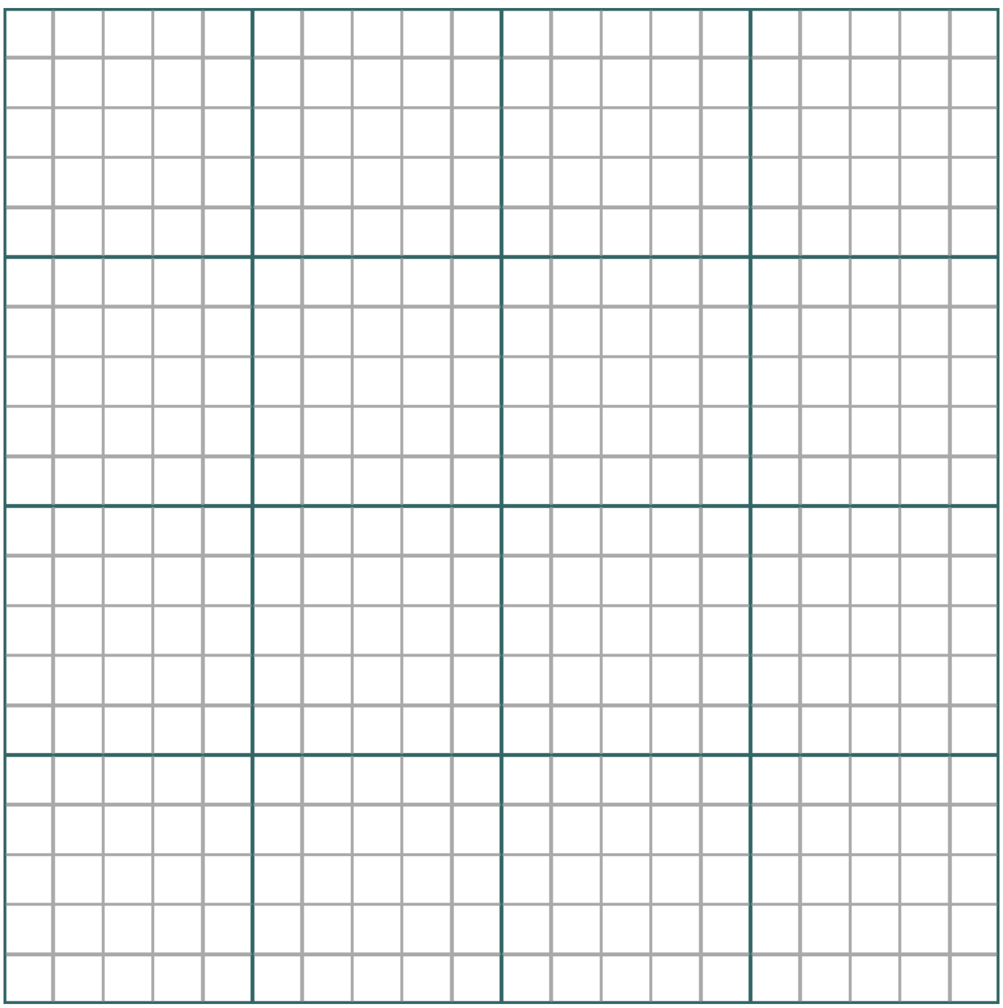


Figure 3. Vector Addition.

- In
$$\mathbf{u} = < u_1, u_2, u_3 >$$
the u_i are called the $\left\{ \begin{array}{l} \textbf{entries} \\ \textbf{components} \\ \textbf{coordinates} \\ \textbf{elements} \end{array} \right\}$ of $\mathbf{u}$.

- The textbook usually uses *components*, but occasionally the word component may mean something else. (For example, we might think of velocity as separated into a component in the direction of motion, and a component perpendicular to that direction.)

- A vector can have any number of components. The set of all vectors with n components is written as $\mathbb{R}^n$. For our purposes, usually $n = 2$ or $n = 3$.

Note that you can think of the set of all real numbers as $\mathbb{R}^1$.

- The **magnitude** of a vector is the length of the arrow that represents it. We write it as
$$\|\mathbf{u}\| = \| < u_1, u_2, u_3 > \| = \sqrt{u_1^2 + u_2^2 + u_3^2}.$$

- The magnitude $\|\mathbf{u}\|$ is also called the $\left\{ \begin{array}{l} \textbf{norm} \\ \textbf{length} \\ \textbf{2-norm} \\ \textbf{Euclidean Length} \\ \textbf{magnitude} \end{array} \right\}$ of $\mathbf{u}$.

- The **zero vector** is the vector with all components equal to zero:

$$\mathbf{0} = < 0, 0 > \qquad \text{or} \qquad \mathbf{0} = < 0, 0, 0 > .$$

$- < 3, 4 >$
$= < -3, -4 >$

- The **negative** of a vector $\mathbf{v}$ is the vector obtained by interchanging the head and tail of $\mathbf{v}$. Algebraically we multiply all components with -1.

-

$$-\mathbf{u} = - < u_1, u_2, u_3 > = < -u_1, -u_2, -u_3 > .$$

- We subtract vectors by adding the negative:

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}).$$

- Clearly

$$\mathbf{u} - \mathbf{u} = \mathbf{0}.$$

The zero vector

$$\mathbf{0} = < 0, 0, 0 >$$

has zero magnitude and **no direction!**

Properties

- Below are some properties of vectors and scalars. You should convince yourself that these are true, going back to the relevant definitions if necessary.

$$
\begin{array}{lrcl}
1. & \mathbf{u}+\mathbf{v} & = & \mathbf{v}+\mathbf{u} \\
2. & (\mathbf{u}+\mathbf{v})+\mathbf{w} & = & \mathbf{u}+(\mathbf{v}+\mathbf{w}) \\
3. & \mathbf{u}+\mathbf{0} & = & \mathbf{u} \\
4. & \mathbf{u}+(-\mathbf{u}) & = & \mathbf{0} \\
5. & a(b\mathbf{u}) & = & (ab)\mathbf{u} \quad = b(au) \\
6. & a(\mathbf{u}+\mathbf{v}) & = & a\mathbf{u}+a\mathbf{v} \\
7. & (a+b)\mathbf{u} & = & a\mathbf{u}+b\mathbf{u} \\
8. & 1\mathbf{u} & = & \mathbf{u} \\
9. & \|a\mathbf{u}\| & = & |a|\mathbf{u}
\end{array}
$$

The textbook does not define the concept of a vector precisely. Mathematically the properties above are used to define vectors. You'll see that definition in Linear Algebra, Math 2250 or 2270.

Unit Vectors

- A vector $\mathbf{u}$ is a **unit vector** if it has magnitude 1:
$$\|\mathbf{u}\| = 1.$$

- If $\mathbf{v}$ is a (non-zero) vector then
$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$
is a unit vector in the same direction as $\mathbf{v}$

- Examples:

$$v = \langle 3, 4 \rangle$$

$$\|v\| = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$u = \frac{1}{5}\langle 3, 4\rangle = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$\left\|\left\langle \frac{3}{5}, \frac{4}{5}\right\rangle\right\| = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{3^2+4^2}{25} = \frac{25}{25} = 1$$

Standard Unit Vectors

- Particularly important unit vectors are those in the directions of the coordinate axes. They have their own special notation.
- in $\mathbb{R}^2$:

$$\mathbf{i} = <1, 0> \quad \text{and} \quad \mathbf{j} = <0, 1> .$$

- in $\mathbb{R}^3$:

$$\mathbf{i} = <1, 0, 0>, \qquad \mathbf{j} = <0, 1, 0>, \quad \text{and} \quad \mathbf{k} = <0, 0, 1> .$$

- Examples:

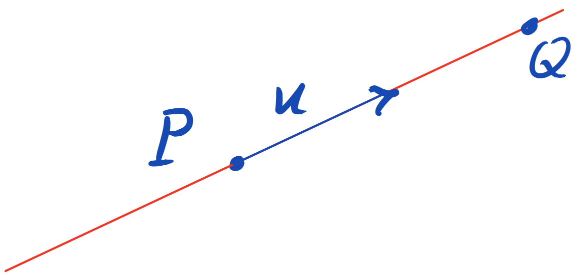
$$<3,4> = 3<1,0> + 4<0,1> = 3i + 4j$$

$$<1,2,-1> = i + 2j - k$$

- The line through the point P in the direction $\mathbf{v}$ can be described by the equation

$$Q = \mathbf{p} + t\cancel{\mathbf{v}}\, u$$

where $\mathbf{p}$ is the vector with tail at the origin and head at P.



Q head of $\overrightarrow{OP} + tu$

$P = (4,5) \qquad u = (1,1)$

$Q(t) = (4+t, 5+t)$

- Example: The diagonals of a parallelogram bisect each other.

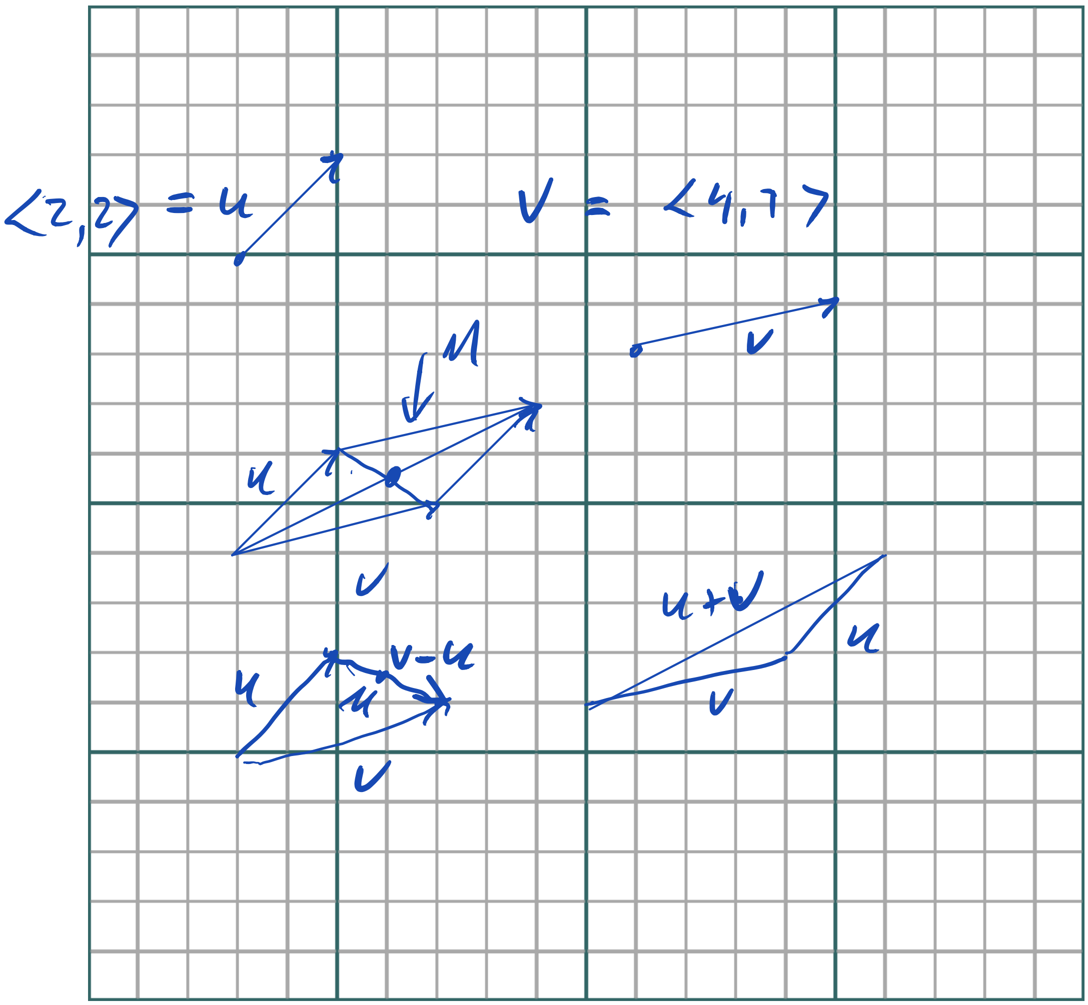


Figure 4. Parallelogram.

$$M = u + \frac{1}{2}(v-u) = u + \frac{1}{2}v - \frac{1}{2}u$$
$$= \frac{1}{2}(u+v)$$