Math 5600 Spring 2021 Notes of 1/19/21

- This course, Math 5600, is our one semester survey of numerical analysis. It is taught every Spring.

- We also teach a two semester introduction, Math 5610-20, of numerical analysis.

   If you plan to take two semesters of numerical analysis you should take the 5610-20 sequence instead of the survey. Taking 5600 and one of 5610 and 5620 will lead to significant overlap while also missing some subjects. We teach 5610 every Fall, and 5620 every Spring. If you want to learn more about numerical analysis after taking the sequence take our graduate courses, Math 6610-20-30.

- You can find a great deal of information about our class on Canvas. Some information, including links to our class notes, is available on our home page

   www.math.utah.edu/~pa/5600

- Go over Syllabus

- Most of the information in this class will come in the form of notes, and classroom discussions and presentation.

- There is no one book that could serve as a satisfactory textbook. However, I will be referencing books frequently throughout the semester, for further reading if you need to learn more about the relevant subject.

- To absorb the material in this class and learn all that’s required for the assignments it should be sufficient to absorb what we’ll discuss in class and what you’ll find in the notes.
There are a great many textbooks on numerical analysis. None stands out. They are all either too idiosyncratic in their coverage, too basic, too theoretical, to recipe focused, to programming focused, or too out-of-date. However, some are excellent in some ways, including the following.

- Kendall E Atkinson, An Introduction to Numerical Analysis, 2nd ed., Wiley, 1987, ISBN 978-0471624899. If you wish to buy a reference book for this class this is the most suitable. It has been used as a textbook in the past. However, it focuses in great depth on what I consider peripheral and distracting issues.


What is Numerical Analysis?

**PP**  Physical Problem. Need not be physics, can be anything for which this mode of investigation is applicable.

**MM**  Mathematical Model. A set of mathematics that describes relevant aspects of PP.

**MQ**  Mathematical Question. A question about MM

**NA**  Numerical Answer. An approximate answer obtained by computer.
• Numerical Analysis encompasses the mathematics of obtaining NA from MQ. Possible Questions include:
  – How do we compute an answer?
  – How can we compute it faster, or more accurately?
  – How can we make use of special structure for special problems?
  – How accurate is the answer? In particular, what is the effect of errors in the problem data, and of errors inherent in the algorithm,

• The first three questions are part of the larger subject of design of algorithms. In my opinion this the exciting part of numerical analysis. The fourth question concerns the traditional area of error analysis, and is sometimes construed as the complete contents of numerical analysis.
Terms

- *Scientific Computing* encompasses the entire chain from PP to NA.

- *Computational (Engineering and) Science* is similar to scientific computing, with an emphasis on computing at the limits of current technology.

- *Applied Mathematics* is used in two senses:
  - Any mathematics that’s not pure mathematics. In that sense, it includes, for example, probability and statistics.
  - The design of mathematical models describing physical phenomena. It’s particularly exciting when the same model describes physically different phenomena (like combustion and the spread of an epidemic).
(Incomplete List of) Specific Subjects

- Course numbers in parentheses indicate which of our courses usually cover this particular subject.

- **Linear Systems** (5600, 5610, 6610)
  
  \[ Ax = b \]

- **Eigenvalue Problems** (5600, 5610, 6610)
  
  \[ Ax = \lambda x \]

- **Interpolation** (5600, 5610, 6610)
  
  \[ p(x_i) = y_i, \quad i = 0, \ldots, n \]

- **Linear Programming** (5600, 5610)
  
  \[ c^T x = \min, Ax \geq 0, x \geq 0 \]

- **Numerical Differentiation** (5600, 5610, 6610)
  
  \[ f'(x) = ? \]

- **Numerical Integration** (5600, 5610, 6610)
  
  \[ \int_{a}^{b} f(x)dx = ? \]

- **Nonlinear Equations** (5600, 5610, 6610)
  
  \[ F(x) = 0, \quad F : \mathbb{R}^n \rightarrow \mathbb{R}^n \]

- **Discrete Approximation** (5600, 5610, 6610)
  
  \[ \|Ax - b\| = \min \]

- **Continuous Approximation** (5600, 5610, 6610)
  
  \[ \|f - \sum_{i=1}^{n} \alpha_i \phi_i\| = \min \]

- **Unconstrained Optimization** (5600, 5610, 5770, 6610)
  
  \[ f(x) = \min \]

- **Constrained Optimization** (5600, 5610, 5770, 6610)
  
  \[ f(x) = \min \text{ plus constraints} \]

- **ODE-IVP** (5600, 5620, 6620)
  
  \[ y' = f(x, y), \quad y(a) = y_0 \]
• **ODE-BVP** (5600, 5620, 6620)
  \[ y''(x) = f(x, y, y'), y(a) = A, y(b) = B \]

• **PDE** (5620, 6620, 6630)
  \[ u_{xx} + u_{yy} = f \]

• **Differential-Algebraic Systems**
  \[ F(x, y, y') = 0, y(a) = y_0 \]

• **Integral Equations**
  \[ y(x) = \int_a^b K(x, t)y(t)dt \]

• **Delay Differential Equation**
  \[ y'(x) + y'(x - 1) = 0 \]

There are a great many connections between these subjects and most lead to a linear system, a linear programming problem, or an eigenvalue problem.