Undergraduate Colloquium

Notes of 8/22/2018

• The undergraduate colloquium is a sequence of weekly talks on mathematical themes that are suitable for undergraduate students. They are open to the public.

• Undergraduate Colloquiums take place every Wednesday, 12:55-1:45, in LCB 225.

• As an undergraduate student, you can take the undergraduate colloquium for credit. You need to attend, missing the presentation no more than twice, and you will need to write a short paper on one of the topics presented during the semester.

• Today we’ll discuss two topics: unsolved and recently solved math problems, and writing a technical paper.

• Note: this is a set of lecture notes, not a technical paper. Don’t emulate these notes in your report.
Famous Simple Unsolved Problems

The Goldbach Conjecture

- This is one of the oldest and best known open problems in mathematics. Its modern version is

  Every even integer greater than 2 can be written as the sum of two prime numbers.

- Nobody knows if this statement is true.

- Prime numbers of course are positive integers that have precisely 2 divisors. The first few are:

  \[2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\]

- Examples

  - Goldbach first stated an equivalent conjecture on June 7, 1742, in a letter to Leonard Euler.
Prime Twins

- a prime twin is a pair of prime numbers whose difference is two.
- Examples:

- It is not known whether there are finitely many or infinitely many prime twins.
Perfect Numbers

- A perfect number is a positive integer that equals the sum of its proper (i.e., positive, excluding the number itself) divisors.
- The first three perfect numbers are:

\[
\begin{align*}
6 &= 1 + 2 + 3 \\
28 &= 1 + 2 + 4 + 7 + 14 \\
496 &= 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248
\end{align*}
\]

- About 300BC Euclid showed that if \(2^p - 1\) is prime then \((2^p - 1) \times 2^{p-1}\) is perfect. (Euclid, Prop IX.36).
- A prime number \(M\) that can be written as \(M = 2^p - 1\) is called a Mersenne Prime Number.
- In the 18th century Leonard Euler proved that all even perfect numbers can be written in the form \((2^p - 1) \times 2^{p-1}\) were \(2^p - 1\) is a Mersenne Prime.
- A worldwide computer search so far has found a total of 50 Mersenne prime numbers. The largest currently (as of August 14, 2018) known Mersenne Prime number is \(M = 2^p - 1\) with

\[p = 77, 232, 917.\]

\(M\) has 23,249,425 decimal digits.
• Thus currently 50 even perfect numbers are known. It is unknown how many even perfect numbers exist, or if there are infinitely many.

• It is also not known if any odd perfect numbers exist.

• You can participate in the search to find more Mersenne primes, and perhaps become famous for finding the largest currently known Mersenne Prime Number. For more info go to

  www.mersenne.org
The Collatz Problem

• Suppose $a_0$ is a positive integer. Consider the sequence

$$a_n = \begin{cases} \frac{a_{n-1}}{2} & \text{if } a_{n-1} \text{ is even} \\ 3a_{n-1} + 1 & \text{if } a_{n-1} \text{ is odd} \end{cases}$$

• Examples:

• Is it true that this sequence reaches 1 at some stage, for all positive integers $a_0$?
• Nobody knows.
• However, the sequence has been tested for all positive integers

$$a_0 \leq 19 \times 2^{58} \approx 5.48 \times 10^{13}$$

• Reference: http://mathworld.wolfram.com/CollatzProblem.html
Millenium Problems

• The Clay Institute (www.claymath.org/) has created a list of seven problems, the Millennium problems, and for each of these problems offers a prize of $1,000,000 to whoever solves it first. (There are rigorous standards for considering a problem “solved”.)

• The problems are too complicated to list here, but here are some interesting additional details:
  – One problem, the “Poincaré Conjecture” was indeed solved, by Grigori Perelman in 2003. In 2006, he was offered the Fields Medal for his accomplishment, and turned it down! In 2010 he was offered the Clay Millenium Prize, and he turned it down!
  – Frank Stenger, formerly of the Departments of Mathematics and Computer Science of the University of Utah, recently submitted a paper that claims to have confirmed the Riemann Hypothesis, another Millenium Problem.
Two Recently Solved Famous Problems

• The Four Color Map Problem. Every proper planar map can be colored with four colors such that no neighboring countries have the same color. This was proved in 1976 by Appel and Haken. The proof required a large amount of computer time, and was controversial at the time. The most comprehensive description of the original calculations is a book with 741 pages:


• Fermat’s Last Theorem, that for any integer \( n > 2 \) there are no three positive integers \( a, b, c \) such that

\[
a^n + b^n = c^n,
\]

was proved by Andrew Wiles in 1994, after working, virtually in secret, for seven years. (If \( n = 2 \) there are infinitely many “Pythagorean Triples” \( (a, b, c) \) satisfying the equation.)
Writing Technical Papers

- No matter what career path you follow, you will have to write: proposals, reports, letters, papers, emails, lecture notes, web pages, perhaps books...
- Your success, reputation, and renumeration, will partially depend on the quality of your written communications.
- It’s not too early to start writing, and being careful and ambitious about it, now!
- To get credit for this colloquium course you will need to write a short report on a topic that is represented during this semester.
- Take this as an opportunity!
Goals

• Your report should be clear.
• It should be engaging.
• Have a brief introduction that describes what you are going to do.
• Otherwise, topics should not require material you have not yet introduced. No forward references!
• Everything should be stated just once. No repetitions!
• Everything should be relevant. No meandering off course!
• Reference your sources.
• Typeset your report!
\textbf{\LaTeX} and \texttt{\LaTeXe}

\textbf{The Past}

\begin{itemize}
\item manuscripts used to be handwritten and then typed by a secretary (or, sometimes, by the author, particularly if the author was a student).
\item The typescript would be submitted to a journal.
\item When accepted it would be typeset by a professional. This process was laborious and error prone.
\item The typeset manuscript would then be proof read by the author. More often than not the author by that time was doing new things and found it hard to concentrate.
\end{itemize}

\textbf{The Present}

\begin{itemize}
\item The author edits the file on a computer using \LaTeX{} or \texttt{\LaTeXe}.
\item The publisher uses the author provided files directly.
\item The process is faster, less error prone, and the author has much more control over his or her paper.
\end{itemize}

\texttt{WYSIWYG} $\leftrightarrow$ \texttt{YGWYW}
For example, the text

\documentclass{article}[18pt]
\begin{document}

The solution of the quadratic equation \[ax^2+bx+c = 0\] (where \((a \neq 0)\)) is given by the \textbf{quadratic formula}:

\[x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.\]

\end{document}

is rendered as

The solution of the quadratic equation

\[ax^2 + bx + c = 0\]

(where \(a \neq 0\)) is given by the \textbf{quadratic formula}:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.\]
Milestones

1978 Donald Knuth of Stanford University releases the first version of \TeX.
1985 Leslie Lamport releases \LaTeX.
1989 Knuth freezes \TeX

What to do

• learn \LaTeX. It is free and available on all Math Department Computers. You can also install and run it on PCs, Macs, and other platforms.
• Use \LaTeX for your report and email me pa@math.utah.edu

a pdf file and the corresponding \LaTeX file, by the last day of final’s week, i.e.,

Friday, December 14, 2018, before midnight
References and Resources

- The home page of our class is

  //www.math.utah.edu/~pa/3000/

- Paul Halmos et al, How to Write Mathematics, AMS publication, ISBN 0-8218-0055-8, 1973. Halmos’ article is the most detailed and instructive of four in that book. You can download the pdf of that article on our home page.

- A sample latex file, sample2e.tex, which can serve as a first introduction to \LaTeX, is available on our home page. You can download that file and experiment by modifying it.

- Too generate the pdf file sample2e.pdf type latex2e sample2e.tex followed by dvi2pdf sample2e


- Donald Knuth, the \TeXbook, Addison-Wesley, 1996, ISBN 0-201-13447-0. The classic reference on plain \TeX.

- Wikipedia pages on \TeX and \LaTeX.