

## Math 1310-1 Notes of January 12, 2016

- As previously announced, I will put copies of my notes online before class. You can see them by clicking on the appropriate date on our home page

<http://www.math.utah.edu/~pa/1310/>

- The notes will contain blank spaces that we will fill in together during class. After class I will replace the online notes with their annotated versions. An asterisk in the calendar will indicate that the replacement has taken place. (Any further modification will be indicated by more asterisks.)
- **Caveat:** In this meeting we will see a lightning fast review of some basic Algebra concepts. The review is meant to remind you of some topics with which you are already familiar. We may not get through this whole set of notes today. In that case, complete them on your own. Lifeng will talk more about Algebra, and in particular some common pitfalls, in the Lab on Thursday.

### Concepts of Algebra

- Algebra differs from arithmetic in that it uses **variables** in addition to numbers.
- A variable is (usually) denoted by an upper or lower case Greek or Roman letter.
- Upper and lower case are distinct! In general

$$x \neq X \tag{1}$$

- **Basic Principle: Variables stand for numbers. If it does not work for numbers it does not work for variables.**

- **Examples:**

- **(Mathematical) Expressions** An expression is a collection of variables and numbers combined by mathematical operations and functions. (Numbers or variables may be absent.)
- **Examples:**

- **Equations:** An equation is of the form

$$\text{Expression 1} = \text{Expression 2} \quad (2)$$

- If there is no equals sign (=) it's not an equation! (It's probably an expression.)
- **Examples:**

- **Evaluating an expression** means to substitute numerical values or expressions for its variables.
- **Examples:**

- **Identities:** An identity is an equation that is true for all values of its variables. (Sometimes there are a few exceptions.)
- **Examples:**

- A set of values of the variables **satisfies an equation** if those values make the equation true.
- **Solving an Equation** means figuring out for which values of its variables satisfy the equation.
- **Examples:**

- **Equivalent Expressions** are expressions that give the same value for all values of their variables.
- **Examples:**



- **Simplifying an Expression** means to replace it with an equivalent expression that has a certain form. The meaning of “simple” depends on the context. Often there is no universal agreement.
- **Examples:**

- **Trigonometry** is the mathematics of angles.
- Angles are measured in radians, i.e., the length of the corresponding arc on the unit circle.
- $\cos x$  and  $\sin x$  are the coordinates of the point

$$(\cos x, \sin x) \tag{3}$$

on the unit circle (centered at the origin with radius 1). As an immediate consequence we have the most important trig identity:

$$\sin^2 x + \cos^2 x = 1. \tag{4}$$

- $\sin x$  and  $\cos x$  are  $2\pi$  periodic and defined for all real numbers  $x$ .
- The tangent of  $x$  is defined by

$$\tan x = \frac{\sin x}{\cos x} \tag{5}$$

- Note that

$$\sin^2 x = (\sin x)^2 \neq \sin x^2 = \sin(x^2). \tag{6}$$

Similarly for  $\cos x$ , of course.

- The **absolute value** of a number  $x$  is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \quad (7)$$

- **Examples:**

- **The Cartesian Coordinate System.** You want to be familiar with it and understand the following terms: horizontal and vertical axis,  $x$ -axis,  $y$ -axis, origin, quadrants (I, II, III, IV), distance, line, slope,  $x$ -intercept,  $y$ -intercept, slope-intercept form, two point form, point-slope form (of a line).

- The **graph of an equation** is the set of all points whose coordinates satisfy the equation.
- **Examples:**

- A formula is an equation that helps you solve certain problems.
- The **Binomial Formulas** are

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\(a - b)^2 &= a^2 - 2ab + b^2 \\(a + b)(a - b) &= a^2 - b^2\end{aligned}\tag{8}$$

- **Examples:**

- The **Summation Symbol**  $\sum$  is defined by

$$\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n \quad (9)$$

- A generalization of the first binomial formula is the **Binomial Theorem**:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} \quad (10)$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (11)$$

- The exclamation mark is pronounced **factorial** and is defined by

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ 1 \times 2 \times \dots \times n & \text{if } n > 0 \end{cases} \quad (12)$$

- **Examples:**

- The **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (13)$$

gives the solution of the **quadratic equations**

$$ax^2 + bx + c = 0. \quad (14)$$

- I recommend you don't bother memorizing it and instead solve any quadratic equation by completing the square.
- **Examples:**



- A **power** or **exponential expression** is of the form  $a^b$  where  $a$  is the **base** and  $b$  is the **exponent**. If  $b = n$  is a natural number  $n$  then  $a^n$  is defined as

$$a^n = \underbrace{a \times a \times \dots \times a}_{n \text{ factors } a}.$$

Everything flows from there. The following properties follow straight from the definition for integer exponents, and for rational and real exponents the definitions are such that these rules (for example) hold:

$$\begin{aligned} a^x a^y &= a^{x+y} \\ \frac{a^x}{a^y} &= a^{x-y} \\ a^x b^x &= (ab)^x \\ (a^x)^y &= a^{xy} \\ a^0 &= 1 (a \neq 0) \end{aligned} \tag{15}$$

- Remember that

$$\sqrt{x} = x^{1/2}, \quad \sqrt[3]{x} = x^{1/3}, \quad \text{and in general} \quad \sqrt[p]{x} = x^{1/p}. \tag{16}$$

- Finally note

$$\sqrt{x^2} = |x| \quad \text{and not } x \text{ in general!} \tag{17}$$