eigenvalues of a symmetric matrix are real

\[ A\mathbf{x} = \lambda \mathbf{x} \]

\[ \overline{A\mathbf{x}} = \overline{\lambda \mathbf{x}} \]

\[ \mathbf{x}^T \overline{A^T} = \overline{\lambda \mathbf{x}}^T \]

\[ \mathbf{x}^T A \mathbf{x} = \lambda \mathbf{x}^T \mathbf{x} \]

\[ \mathbf{x}^T A^T \mathbf{x} = \overline{\lambda} \mathbf{x}^T \mathbf{x} \]

\[ \mathbf{x}^T A \mathbf{x} = \overline{\lambda} \mathbf{x}^T \mathbf{x} \]

\[ \lambda \mathbf{x}^T \mathbf{x} = \overline{\lambda} \mathbf{x}^T \mathbf{x} \]

\[ \lambda = \overline{\lambda} \]

\[ (A - \lambda I) \mathbf{x} = 0 \]
\[ A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 5 \\ 6 & 4 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} = u \]

\[ L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 5 \\ 6 & 4 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \]

**Pivoting**

\[ \xi \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Ref
permutation matrix

obtained from

I

by permuting

rows

\[
\begin{bmatrix}
0 & 1 \\
1 & 0 \\
2 & 1 \\
3 & 1
\end{bmatrix} \quad \begin{bmatrix}
2 & 1 \\
4 & 3
\end{bmatrix}
\]

\[
0 & 1 \\
1 & 0
\]

Ex.: \( P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

\[ p^T p = I \]

\[ PA = LU \]

\[ A = P^T L U \]

\[ p(x) = x^T A x \quad A = A^T \quad n \times n \]

\[ n = 1 \quad A = \begin{bmatrix} a_1 \end{bmatrix} \quad p(x) = a x^2 \]

\[ F: \mathbb{R}^n \rightarrow \mathbb{R} \]

\[ F(x) = F(a) + \nabla F(a)^T (x - a) + \frac{1}{2} (x - a)^T \nabla^2 F(a) (x - a) + \ldots \]

\[ \nabla F = \begin{bmatrix} \frac{\partial F}{\partial x_i} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix} \quad \nabla^2 F = \begin{bmatrix} \frac{\partial^2 F}{\partial x_i \partial x_j} \\ \vdots \\ \frac{\partial^2 F}{\partial x_n \partial x_i} \end{bmatrix} \]
\[ A = A^T \quad \text{for all } x \neq 0 \quad 2 \text{ value} \]
\[ x^T A x > 0 \quad \text{A positive definite for all } x > 0 \]
\[ < 0 \quad \text{negative definite for all } x < 0 \]
\[ > 0 \quad \text{positive semidefinite for all } x > 0 \]
\[ \leq 0 \quad \text{negative semidefinite for all } x \leq 0 \]

\[ \text{some } > 0, < 0 \quad \text{ indefinite} \]
\[ \text{some } \lambda > 0 \]
\[ \text{some } \lambda < 0 \]

\[ A = A^T \]
\[ \begin{bmatrix} 5 & 1 & 1 \\ 1 & 6 & 3 \\ 1 & 3 & 5 \end{bmatrix} \]

\[ \text{Finite Diff Eq.} \]
\[ \sum_{i=0}^{k} \phi_i x^p_{n+i} = 0 \quad \forall n = 0, 1, 2, \ldots \]

homogeneous

\[ p(x) = \sum_{i=0}^{k} \phi_i \cdot r^i \]
\[ \Phi_m = r^m \]
\[ P(x) = 0 \]
\[ y_m = r^m \]
Fibonacci:

\[ x_{n+2} - x_{n+1} - x_n = 0 \quad x_0 = x_1 = 1 \]

\[ p(r) = r^2 - r - 1 = 0 \]

\[ r = \frac{1 \pm \sqrt{1+4}}{2} \]

\[ r_+ = \frac{1 + \sqrt{5}}{2} \]

\[ r_- = \frac{1 - \sqrt{5}}{2} \]

\[ x_n = A r_+^n + B r_-^n \]