6.6 Applications

- Work is the integral of force with respect to distance.

- Quick Review:

- Suppose we move along the $x$-axis from $x = a$ to $x = b$ and the force is $f(x)$, where $f$ is a continuous function. Partition $[a, b]$ into subintervals $[x_{i-1}, x_i]$ where

$$a = x_0 < x_1 < \ldots < x_n = b.$$  

Pick a sample point $x_i^*$ in each subinterval $[x_{i-1}, x_i]$. Approximate the work in that subinterval by assuming the force is constant, and equal to $f(x_i^*)$, in that interval. You get

$$W \approx \sum_{i=1}^{n} f(x_i^*) \Delta x.$$  

Then

$$W = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_{a}^{b} f(x)dx.$$
Example 4: A tank has the shape of an inverted circular cone with height 10m and base radius 4m. It is filled with water to a height of 8m. Find the work required to empty the tank by pumping all the water to the top of the tank. The density of water is 1000kg/m$^3$.

Note: displacement is measured in meters, force in Newtons, work in Newton meters, gravity on earth is 9.8m/second squared, 1 Newton is the force required to accelerate 1kg by 1m/second squared. Thus a mass of 1kg on the surface of the earth has a weight (force) of 9.81 Newtons.
Cool Example: Compute the work required to lift an object from the surface of the earth to infinity. Gravity causes an acceleration of 9.81 meters per second squared on the surface of the earth, and gravity is inversely proportional to the distance from the center of the earth. The radius of the earth is 6371 km. Kinetic energy is $\frac{1}{2}mv^2$. The escape velocity is that velocity for which the kinetic energy equals the work required to lift an object to infinity. Compute the escape velocity on a body with radius $R$ and surface gravity $g$. 
The table shows escape velocities from the surface of solar system objects.

- On the other hand, the escape velocity from the sun at the location of the earth is 42.1 km/s, and the escape velocity from the milky way at the location of the solar system is about 500 km/s.
Problem 16, page 473. A 10-foot chain weighs 25 pounds and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it’s level with the upper end.