Math 1310-4 Notes of 11/25/19

Semester End

• Here is a summary for the final couple of weeks of our semester.
  – hw 14 is now open. Its main purpose is to serve as a review of the whole semester. It will close Wednesday, 10/04/12, 1 minute before midnight.
  – this short week: talk more about Chpt 6
  – next week, Mo-We and Th Lab: Review
  – Friday, 12/6, 10:30am-12:30pm, Q&A, HEB 2004
  – Monday, 12/9, 10:30am-12:30pm, HEB 2004, Final Exam, comprehensive, format like midterms.

• Next semester, Math 1320 will start in Chpt 6, and run through Chapter 11 of our current textbook.
6.2-3 Volumes

- We saw last week that the basic idea of a volume computation is to integrate the area of a cross section in a direction perpendicularly to the cross section.

- Definition, page 439, textbook. Let $S$ be the solid that lies between $x = a$ and $x = b$. If the cross-sectional areas of $S$ in the plane $P_x$ through $x$ and perpendicularly to the $x$-axis use $A(x)$, where $A$ is a continuous function, then the volume of $S$ is

$$V = \lim_{n \to \infty} \sum_{j=1}^{n} A(x_i) \Delta x = \int_{a}^{b} A(x) \, dx.$$ 

- This is just like computing an area of a region by integrating the length of the cross section in a direction perpendicularly to the cross section.

- The textbook has some fancy pictures illustrating the concept.
Example 8. Compute the volume of a right square pyramid of height $h$ and side length $L$. Before you start, think about your expectation.

$$A(\Delta) = \frac{1}{2} b L$$

$A_0 = L^2$ area of base

$$\frac{L}{h} = \frac{2Y}{x} \quad y = \frac{xL}{2h}$$

$$A(x) = 4y^2$$

$$V = \int_0^h 4y^2 \, dx = \int_0^h 4 \frac{2L^2}{4h^2} \, dx$$
\[
\frac{L^2}{h^2} \int_0^L x^2 \, dx
\]

\[
= \frac{L^2}{h^2} \left( \frac{x^3}{3} \right)_0^L
\]

\[
= \frac{L^2}{h^2} \left( \frac{L^3}{3} \right) = \frac{L^2 h}{3}
\]

\[
= \frac{1}{3} A_0 h
\]
General Cones

- Start with a region $R$ in the plane. Suppose its area is $A$. Add a point $P$ above the region, at a height $h$. Connect $P$ to every point on the boundary of $R$. You get a generalized cone. The process is called coning.

The volume of your general cone equals

$$V = \frac{Ah}{3}$$

independent of the shape of $R$!
Example 1, page 440, textbook. Show that the volume of a sphere of radius \( r \) is \( \frac{4}{3} \pi r^3 \). Do it two different ways.
maximize the volume of the cylindrical plug

\[ V = \frac{1}{3} \pi r^2 h \]

\[ h = \sqrt{R^2 - r^2} \]

\[ V = \frac{2 \pi r^2 \sqrt{R^2 - r^2}}{3} \]

\[ V = \frac{2 \pi}{3} \left( 2r \sqrt{R^2 - r^2} + r^2 \left( -\frac{2r}{2 \sqrt{R^2 - r^2}} \right) \right) \]

\[ V = 0 \]

\[ \frac{2 \pi}{3} \times \sqrt{R^2 - r^2} \]

\[ 2r(R - r^2) = r^3 \]

\[ 2 (R^2 - r^2) = r^2 \]

\[ 2R^2 = 3r^2 \]

\[ r = \frac{2}{3} R \]
\[ r = \frac{2}{3} \sqrt{R^3} \]

Shell:

\[ A_{\text{shell}} = 2 \pi \int_{0}^{R} \sqrt{R^2 - x^2} \, dx \]

Volume of the spherical segment:

\[ V_{\text{spherical}} = 2 \left[ \int_{0}^{R} 2\pi x \left( \sqrt{R^2 - x^2} \right)^{3/2} \, dx \right] \]

\[ = 4\pi \left( \frac{1}{2} \frac{2}{3} \left( R^3 - x^2 \right)^{3/2} \right) \bigg|_{0}^{R} \]

\[ = \frac{4}{3} \pi \left( R^3 \right)^{3/2} = \frac{4}{3} \pi R^3 \]

\[ \frac{4}{3} \pi \left( R^3 - r^3 \right) \]
• Problem 38, page 454, textbook. You drill a hole of radius $r$ through a sphere of radius $R$, such that the axis of the whole passes through the center of the sphere. Compute the volume of the part of the sphere that is left. The interior of the hole forms a cylinder. How do you pick $r$ so as to maximize the volume of that cylinder?