Review

\[ f(t) = A_0 a^t \quad \text{if} \quad a > 0 \quad a = e^{\ln a} \]

\[ = A_0 (e^{\ln a})^t \quad a \neq 1 \quad \text{a base} \]

\[ = A_0 e^{(\ln a)t} \quad t \text{ time} \]

\[ = A_0 e^{kt} \quad A_0 = \text{initial value} \]

\[ k = \ln a \]

\[ f'(t) = A_0 e^{kt} \]

\[ = k A_0 e^{kt} \]

\[ = kf(t) \]

\[ a > 1, \quad k > 0 \quad \text{exponential growth} \]

\[ a < 1, \quad k < 0 \quad \text{exponential decay} \]

\[ f(t+T) = A_0 e^{k(t+T)} = e^{kT} A_0 e^{kt} \]

\[ T \text{ constant} \quad = e^{kT} f(t) \]

\[ \text{Doubling time} \ T \quad e^{kT} = aT \]

\[ f(t+T) = 2f(t) = e^{kT} f(t) \]
\[ e^{kT} = 2 \]

\[ kT = \ln 2 \]

\[ T = \frac{\ln 2}{k} \]

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Doubling time \( T \)

**Effective annual interest rate is 5%**

\[ a = 1.05 \]

\[ 1.05^T = 2 \]

\[ T \ln 1.05 = \ln 2 \]

\[ T = \frac{\ln 2}{\ln 1.05} \]

\[ T \approx 14.2 \]

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\(^{14}\text{C} \quad 5730 \text{ years}\)

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22% \( \frac{^{14}\text{C}}{\text{C}} \) in a sample of leather

when was the leather made

\[ a^T = \left( \frac{1}{2} \right)^{\frac{T}{5730}} = 0.22 \]

\[ T = ? \]
\[ T = \frac{\ln 0.22}{\ln 0.5} \cdot 5730 \]

\[ f(t) = \left( \frac{1}{2} \right)^{5730/2} = a^t = e^{kt} \]

\[ k = \ln a = \ln \left( \frac{1}{2} \right) \]

\[ DT = 66 \text{ years} \]

\[ f(t) = 2^{-t/66} \quad a = 2 \]

Tripling time 15 years

\[ f(t) = 3^{t/15} \]

Half-life 7 years

\[ f(t) = \left( \frac{1}{2} \right)^{t/7} = 2^{-t/7} \]
\[ a_n = \left( \frac{1}{2} \right)^{n/7} \quad a_{n+1} = \left( \frac{1}{2} \right)^{n/7} \cdot a_n \]

\[
\sum_{n=0}^{\infty} a_n = \frac{1}{1-r} = \frac{1}{1-\left(\frac{1}{2}\right)^{1/7}} \quad \text{WTT}
\]

- \( p \%) \text{ interest per month} \)
- add \( A \) each month \( a = 1 + \frac{p}{100} \)
- how much do you have after \( n \) months

\[
A \cdot a^n + A \cdot a^{n-1} + A \cdot a^{n-2} + \ldots + A \cdot a + A = A \left( \sum_{k=0}^{n} a^k \right) = A \cdot \frac{1-a^{n+1}}{1-a}
\]

\[
A = 1 \quad p = 0.5
\]

\[
A = 1 + \frac{0.5}{100} = 1.005
\]

\( n = 600 \) months

Total Investment is 600

Total savings = \( \frac{1-a^{601}}{1-a} \)