Reminders

• Will let you know your Exam 3 score when done grading.

• Last hw (s14) opened today. Its main purpose is to help you review the semester.

• DS today, 3:00, tomorrow, 5:00, and Wednesday 5:00.

• No DS on Thanksgiving or Friday this week.

• Will finish Chpt 10 this week.

• Mo, Tu, We after Thanksgiving is review of whole semester.

• Final Exam December 11-12. Format as midterms, except it will cover the whole semester and be twice as long.
10.5 Polar Coordinates

- **Cartesian Coordinates** describe the location of a point by its projections onto the $x$ and $y$ axes.

- You say how far you need to go North or South, and East or West, starting at the origin.

- You could also describe the location of a point by giving its distance and direction.

- The parking lot is one mile Northeast of here ...

- That’s what **polar coordinates** do.

- You need a point, the **origin** and a half line starting at the origin, the **polar axis**.

- In Geography the polar axis points north, in polar coordinates traditionally it is the positive part of the $x$ axis.

- You describe the location of a point by the angle $\theta$ that the line to it makes with the $x$-axis, and the distance, usually denoted by $r$. We write the point as $P(r, \theta)$.

2 It is sometimes convenient to allow negative values of $r$, which indicates that we go into the opposite direction. Suppose $r > 0$. Then

$$P(-r, \theta) = P(r, \theta + \pi)$$
• For example, going 1/2 miles north is the same as going negative 1/2 miles south.

\[ P(0.5, \frac{\pi}{2}) = P(-0.5, \frac{3\pi}{2}) = P(-0.5, \frac{7\pi}{2}) \]

**Figure 1.** Unit Circle.
Major Fact: While a point determines its cartesian coordinates uniquely, the polar coordinates of a point are not unique!

- Example

\[ P(1, 45^\circ) = P(1, 405^\circ) = P(-1, -135^\circ) = P(-1, 225^\circ). \]

**Figure 2.** Unit Circle.
Coordinate Conversion

Figure 3. Unit Circle.

- We want to be able to go back and forth between polar and cartesian coordinates.

\[(x, y) \leftrightarrow P(r, \theta)\]

- We have the equations

\[x = r \cos \theta \quad \text{and} \quad y = r \sin \theta\]

which implies that

\[x^2 + y^2 = r^2(\cos^2 \theta + \sin^2 \theta) = r^2.\]
\[ 0 \leq \theta \leq 2\pi \]
\[ \theta = \arctan \frac{y}{x} \]

- this implies that

\[ \frac{y}{x} = \frac{r \sin \theta}{r \cos \theta} = \tan \theta. \]

\[ \begin{array}{c}
\text{But computing } \theta, \text{ given } x \text{ and } y, \text{ is not as simple as applying the inverse tangent function.}
\end{array} \]

\[ \text{Figure 4. Graph of } y = \arctan x. \]

- It can’t possibly work in general since \( \theta \) is an angle in \([0, 2\pi]\), say, whereas the inverse tangent function returns an angle in \([\frac{-\pi}{2}, \frac{\pi}{2}]\).

- Not only do the intervals have different centers, they also have different lengths!
Figure 5. Unit Circle.

\[ \theta = \pi + \arctan \frac{y}{x} \]

\[ \theta = \arctan \frac{y}{x} \]

\[ \theta = 2\pi + \arctan \frac{y}{x} \]
Polar Equations

- A **polar equation** is an equation involving $r$ and $\theta$.

- The **graph** of a polar equation is the set of all points $P(r, \theta)$ that satisfy the equation.

- Example:
  
  $$r = \text{constant}$$
Example:

\[ \theta = \text{constant} \]
• Example:

\[ r = \theta \]
Figure 6. Spiral, $r = \theta$. 

Example:

\[ r = \sin \theta, \quad 0 \leq \theta \leq 2\pi. \]

\[ (x-h)^2 + (y-k)^2 = r^2 \]
Figure 7. shifted circle, $r = \sin \theta$. 
• Example:

\[ r = \frac{1}{1 - \cos \theta}, \quad 0 \leq \theta \leq 2\pi. \]

\[ r - r \cos \theta = 1 \]

\[ r = 1 + r \cos \theta = 1 + x \]

\[ r^2 = x^2 + y^2 = (1 + x)^2 = x^2 + 2x + 1 \]

\[ y^2 = 2x + 1 \]

\[ x = \frac{1}{2} (y^2 - 1) \]
Figure 8. Parabola, $r = \frac{1}{1-\cos \theta}$. 
Figure 9. Parabola close to origin.
• Let’s compute the polar equation of a line.

\[
\frac{d}{r} = \cos(\theta - \theta_0) \\
\tau = \frac{d}{\cos(\theta - \theta_0)}
\]
Figure 10. Line, \( r = \frac{1}{\cos(\theta - 1)} \).