Announcements

- We are done with chapter 9.
- Today will start with chapter 10. However, next week we will have three days of review of chapter 9, and on Friday and Saturday we’ll have exam 3. It will contain 8 questions, and the format will be as usual.
- Home work 12 is mostly on conic sections, and is very short because of the upcoming exam.
- However, the last problem on that set is meant to help you prepare for the exam. You definitely want to finish that problem before next Friday!
- We will have class on 11/20, but we will cover no new material. Liz and I will answer questions on anything other than chapter 9, presumably mostly about conic sections.
- DSs today, 1:00, 2:00, 3:00, and Sunday, 7:00.
10.1-10.3 Conic sections

• First covered in College Algebra, and not a prerequisite for anything else we will do in 1220 or 2210.

• So we will be brief.

• The textbook has an unusual approach to conic sections. It defines everything in terms of a focus and a directrix.

• Our quick review will be more conventional.

• Why “conic section”?
**Figure 1.** Conic Sections.

Cones are infinite

Figure 1 is from the Wikipedia page on Conic Sections.

- Geometrically, a conic section is the intersection of a plane and a double cone.

- Algebraically, it is the set of all points satisfying a quadratic equation in two variables:

  \[ Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad (1) \]

  where \( A, B, C, D, E, F \) are given real coefficients.

- You can see how to obtain that equation. We
have not talked about equations in three-dimensional space, but the equation

\[ z^2 = x^2 + y^2 \]  \hspace{1cm} (2)

defines a double cone, and the equation

\[ z = ax + by + c \]  \hspace{1cm} (3)

defines a plane.

- Substituting (3) in (2) gives the equation

\[ z^2 = (ax + by + c)^2 = x^2 + y^2 \]

which can be easily converted to the form (1).
• We also have more specific geometric definitions:

• A **circle** is the set of all points that have the same distance (the **radius**) from a given point (the **center**).

\[(x - h)^2 + (y - k)^2 = r^2\]
• An **ellipse** is the set of all points for which the **sum** of the distances to two given points (focus, focuses, foci) is constant.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
- A **hyperbola** is the set of all points for which the **difference** of the distances to two given points is constant.

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
A **Parabola** is the set of all points for which the distances from a given point (the **focus**) and a given line (the **directrix**) are equal.

\[ y^2 = 4px \]
• Return to the equation

\[ Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \]

• One can rotate the coordinate axes so that \( B = 0 \). Then we get

\[ Ax^2 + Cy^2 + Dx + Ey + F = 0 \]

• For particular values of \( A, C, D, E, F \) We can determine the type of conic section we can define the type of conic section defined by the equation by completing the square.

• For example, consider

\[ x^2 + 4x + y^2 + 6y = 0. \]

\[ \begin{align*}
  x^2 + 4x + y^2 + 6y + 9 &= 13 \\
  (x+2)^2 + (y+3)^2 &= \left(\sqrt{13}\right)^2 \\
  (x-h)^2 + (y-k)^2 &= r^2
\end{align*} \]

\[ x + 2 = x - h \]

\[ h = -2 \]
\[2x^2 + 4y^2 + 2y = 0\]

\[2x^2 + 4\left(y^2 + \frac{y}{2} + \frac{1}{16}\right) = \frac{1}{4}\]

\[\left(y + \frac{1}{4}\right)^2\]

\[8x^2 + 16\left(y + \frac{1}{4}\right)^2 = 1\]

\[\frac{x^2}{\frac{1}{8}} + \frac{(y + \frac{1}{4})^2}{\frac{1}{16}} = 1\]

\[a = \sqrt{\frac{1}{8}}, \quad b = \frac{1}{4}\]
\[ x^2 + 4x + y = 0 \]

\[
\gamma = -x^2 - 4x = -(x^2 + 4x + y) + y
\]

\[
\gamma - y = \bigcirc (x - 2)^2
\]
There are some degenerate types of “conic sections”. Some Examples:

\[ x^2 + y^2 = -1 \]
\[ x^2 + y^2 = 0 \]
\[ Dx + Ey + F = 0 \text{ where } |D| + |E| \neq 0. \]
\[ x^2 - 1 = 0 \]
\[ x^2 = y^2 \]
\[ 0 = 0 \quad \text{entire plane} \]

\[
\frac{5}{2} \int \sin x^2 \, dx = \infty \quad \int_{\infty}^{p} \frac{1}{x^p + 1} \, dx
\]

\[ f(x) = \sin x^{1/2} \]

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots
\]

\[
\sin^{1/2} x = x^{1/2} - \frac{x^{3/2}}{3!} + \frac{x^{5/2}}{5!} - \frac{x^{7/2}}{7!} + \cdots
\]

\[
\int \sin^{1/2} x \, dx = \left( \frac{2}{3} \right) x^{3/2} - \left( \frac{2}{5} \right) x^{5/2} + \left( \frac{2}{7} \right) x^{7/2} - \cdots
\]

\[
\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n
\]

Maclaurin of \( \sin x^2 \) ?

MacLaurin of \( \sin x^2 \) ?
\[
\int_{0.5}^{1.5} \sin(\sqrt{x}) \, dx
\]

\[f(x) = x^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!} (x-1)^n\]

\[f(1) = 1 = 1\]

\[f'(x) = x^x (\ln x + 1) \quad f'(1) = 1\]

\[f''(x) = x^x (\ln x + 1)^2 + x^x \frac{1}{x}\]

\[f''(1) = 2\]

\[f''(x) = x^x \left[ (\ln x + 1)^2 + \frac{1}{x} \right]\]

TS \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n

\[\lim_{n \to \infty} a_{2n} = 1 \quad \lim_{n \to \infty} a_{3n} = 1\]
\[
\lim_{n \to \infty} a_n = L
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 \\
\end{array}
\]

\[a_n = \begin{cases} 
1 & \text{if } n \text{ is divisible by 2 or 3} \\
2 & \text{else}
\end{cases}\]