Announcements

• You should have received an email from me with login info for WeBWorK. The first homework is open, and serves as a review of Math 1210.

• hws open on Mondays and close 12 days later on Fridays. However, you should finish each homework before the next one opens.

• Thus you should finish hw 1 by Sunday. However, it will be open until a week from today.

• hw 2 will open Monday.

• The key to succeeding in mathematics is to appreciate that it builds on itself. Stay on top of things and don’t fall behind!

6.1 The natural logarithmic function

• We are now starting with the subject of Math 1220 proper.

• I assume you are familiar with exponentials and logarithms, particularly the natural logarithm.

\[ \ln, \quad e^x, \quad e = 2.71828\ldots \]
The major fact that we will begin to understand today is

\[ \frac{d}{dx} \ln x = \frac{1}{x} \]

or, equivalently,

\[ \int_1^x \frac{1}{t} \, dt = \ln x. \]

**Figure 1.** $\ln$ and its derivative.
• This makes geometric sense, as illustrated in Figure 1.

• Note that our power rule

\[ \int t^r \, dt = \frac{t^{r+1}}{r+1} \]

fails if \( r = -1 \). We will see today that

\[ \int_1^x t^{-1} \, dt = \int_1^x \frac{1}{t} \, dt = \ln x. \]

• There are many different ways to introduce the derivatives of exponentials and logarithms. In the spirit of Math 1210 we could define

\[ \frac{d}{dx} \ln x = \lim_{h \to 0} \frac{\ln(x + h) - \ln x}{h}. \]

• It would be hard to compute that limit with what we have learned in Math 1210.

• Instead the textbook proceeds as follows: It defines

\[ \ln x = \int_1^x \frac{1}{t} \, dt. \]

• With that definition it follows immediately from the FToC that

\[ \frac{d}{dx} \ln x = \frac{1}{x}. \]
• Of course, this definition is useful only if it is consistent with our already existing understanding of the natural logarithm!

• It is, and we will eventually recognize that this is so by showing that the function we defined has all the properties if the natural logarithm with which we are familiar.

• First example:

\[
\frac{d}{dx} \ln |x| = \frac{1}{x}.
\]
• Next verify that the following properties of the natural logarithm are satisfied.

\[
\begin{align*}
\ln 1 &= 0 \quad (i) \\
\ln ab &= \ln a + \ln b \quad (ii) \\
\ln \frac{a}{b} &= \ln a - \ln b \quad (iii) \\
\ln a^r &= r \ln a \quad (iv)
\end{align*}
\]

(assuming that \(a, b > 0\)).

\[(i) \quad \ln 1 = 0\]
(ii) \( \ln ab = \ln a + \ln b. \)
\[(iii) \quad \ln \frac{a}{b} = \ln a - \ln b.\]
(iv) \[ \ln a^r = r \ln a. \]
Examples

Compute \[ \frac{d}{dx} x^2 = ? \]
in two different ways.
\[ \int \frac{2x}{1 + x^2} \, dx = ? \]
\[
\frac{d}{dx} x^x = ?
\]
\[ \int \tan x \, dx = ? \]
\[ \int \frac{x^2 + 1}{x + 1} \, dx = ? \]