1. Use the product and quotient rules to differentiate the following functions. No need to simplify.

(a) \( D_x (x^2 \sin x) = (2x) \sin x + x^2 \cos x \)

(b) \( D_x \left( \frac{x^3-x}{x^2+3} \right) = \frac{(x^2+3)(3x^2-1)-(x^3-x)(2x)}{(x^2+3)^2} \)

(c) \( D_x (x^5 \sin x \cos x) = 5x^4 \sin x \cos x + x^5 \cos^2 x - x^5 \sin^2 x \)

(d) \( D_x \left( \frac{x^3 \cos^2 x}{1-\sin x} \right) = \frac{(1-\sin x)(3x^2 \cos x - x^3 \sin x) - (x^3 \cos x)(-\cos x)}{(1-\sin x)^2} \)

(e) \( D_x \left( \left( x^4 + x \right) \left( \frac{\sin x - x}{\cos x} \right) \right) = (4x^3 + 1) \left( \frac{\sin x - x}{\cos x} \right) + (x^4 + x) \left( \frac{(\cos x)(\cos x - 1) - (\sin x)(\sin x)}{\cos^4 x} \right) \)

2. Let \( f(x) = \frac{x^2 + 1}{x + 1} \)

Compute the fourth derivative of \( f \).

Conceptually, the most straightforward approach to solving this problem is to apply the quotient rule four times. When doing this it is crucial to simplify the expression after computing each derivative. It is also convenient to write the second derivative as a power with a negative exponent and then apply the power rule. We obtain:

\[
\begin{align*}
f(x) &= \frac{x^2 + 1}{x + 1} \\
f'(x) &= \frac{2x(x+1) - (x^2 + 1)}{(x+1)^2} = \frac{x^2 + 2x - 1}{(x+1)^2} \\
f''(x) &= \frac{2(x+1)^2 - 2(x^2 + 2x - 1)2(x+1)}{(x+1)^4} \\
&= \frac{4}{(x+1)^3} \\
f'''(x) &= -12(x+1)^{-4} \\
f''''(x) &= 48(x+1)^{-5}
\end{align*}
\]

However, the differentiation becomes even simpler if we first divide the numerator by the denominator using long division with remainder. This gives

\[
f(x) = x - 1 + \frac{2}{x+1} = x - 1 + 2(x+1)^{-1}
\]

Differentiating this expression four times gives the much simpler calculation

\[
\begin{align*}
f(x) &= x - 1 + 2(x+1)^{-1} \\
f'(x) &= 1 - 2(x+1)^{-2} \\
f''(x) &= 4(x+1)^{-3} \\
f'''(x) &= -12(x+1)^{-4} \\
f''''(x) &= 48(x+1)^{-5}
\end{align*}
\]
which of course is the same answer as before.

3. For this problem, let

\[ f(x) = x^2 \quad g(x) = \sin x \quad h(x) = \frac{1}{x} \quad (1) \]

For each of the functions listed below, determine how to write it as a composition of these functions, circle the correct composition, and then use the chain rule to differentiate it.

- **Circle one:** \(f(g(x))) g(f(x)) g(h(x)) h(g(x)) f(h(x)) h(f(x))\)
  \[ F'(x) = 2 \sin x \cos x \]

- **Circle one:** \(f(g(x)) g(f(x)) \boxed{g(h(x))} h(g(x)) f(h(x)) h(f(x))\)
  \[ G'(x) = \cos \left( \frac{1}{x} \right) \left( \frac{-1}{x^2} \right) \]

- **Circle one:** \(f(g(h(x)))) g(f(h(x))) \boxed{h(g(x))} f(h(g(x)))\)
  \[ H'(x) = -2(\sin x)^3 \cos x \]

- **Circle one:** \(f(g(h(x)))) \boxed{g(f(h(x)))} h(g(f(x))) f(h(g(x)))\)
  \[ J'(x) = 2 \sin \left( \frac{1}{x} \right) \cos \left( \frac{1}{x} \right) \left( \frac{-1}{x^2} \right) \]

- **Circle one:** \(f(g(h(x)))) \boxed{g(f(h(x)))} h(g(f(x))) f(h(g(x)))\)
  \[ K'(x) = \cos \left( \frac{1}{x^2} \right) \left( -2x^{-3} \right) \]

(a) Two of the functions in (1) have the property that they result the same function when composed in either order (for example, perhaps \(f(g(x)) = g(f(x))\)). Which functions from (1) have this property?

\(f(h(x)) = h(f(x)) = \frac{1}{x^2}\)