-1- (Language.) Define what we mean by a cubic polynomial. Give a verbal definition and a formula.

Discussion:
A cubic polynomial is a polynomial of degree 3. It can be written as
\[ f(x) = ax^3 + bx^2 + cx + d. \]

-2- (Logic.) Consider the (true) statement: All differentiable functions are continuous functions. State its converse, its negation, and its contrapositive.

One way to put the converse is: All continuous functions are differentiable. (Of course, this statement is false.)

Possible statements of the negation include: There is a discontinuous differentiable function. Some differentiable functions are discontinuous. Not all differentiable functions are continuous. (All of these statements are of course false.)

Possibilities for the (true) contrapositive statement include: If a function is discontinuous it is not differentiable. No discontinuous function is differentiable. All discontinuous functions are non-differentiable.

-3- (Equation Solving.) Find all solutions of the equation
\[ x^4 - 5x^2 + 6 = 0 \]
This is an equation of quadratic type that you can turn into a quadratic equation by substituting \( z = x^2 \). It is perhaps more straightforward to factor it right away:
\[ x^4 - 5x^2 + 6 = (x^2 - 2)(x^2 - 3) = 0 \]
Thus \( x^2 = 2 \) or \( x^2 = 3 \) and the four solutions of the original equation are \( x = \pm\sqrt{2} \) and \( x = \pm\sqrt{3} \).

-4- (Graphing.) Draw the graph of the function \( f(x) = (x - 2)^2 + 3 \)

This is the parabola \( y = x^2 \) shifted 2 units to the right and 3 units up. The graph is shown in Figure 1.

-5- (Line.) Find the intersection of the two lines \( y = 2x + 3 \) and \( y = -x + 9 \)
We equate the two right hand sides
\[ 2x + 3 = -x + 9 \]
and solve for \( x = 2 \). Then we substitute that value in the equation of the first line to get \( y = 2 	imes 2 + 3 = 7 \). To check the answer we also substitute in the second equation to get the same value \( y = -2 + 9 = 7 \). The intersection is \((2, 7)\).

-6- (Limits.) Compute
\[
\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}.
\]
This is of course the derivative of \( f(x) = \frac{1}{x} \) which we computed in class as \( f'(x) = -\frac{1}{x^2} \). That would be a fine answer, but you can also redo the calculation we did in class:
\[
\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)h} = \lim_{h \to 0} \frac{-h}{x(x+h)} = \lim_{h \to 0} \frac{-1}{x} = -\frac{1}{x^2}.
\]

-7- (Continuity.) Let
\[
f(x) = \begin{cases} 
\frac{x^2 - 4}{x+2} & \text{if } x \neq -2 \\
\frac{k}{2} & \text{if } x = -2 
\end{cases}
\]
For what value of \( k \) is \( f \) continuous?
We get
\[
\lim_{x \to -2} \frac{x^2 - 4}{x+2} = \lim_{x \to -2} \frac{(x-2)(x+2)}{x+2} = \lim_{x \to -2} x - 2 = -4.
\]
Thus we choose \( k = -4 \) to get a continuous function.

-8- (A Word Problem.) You have a triangle where each side has the same length 1. Compute its altitude.

The altitude (line segment from one vertex to the midpoint of the base) forms one short side of a right triangle with a base of length 2 and a second short side of length \( \frac{3}{2} \). This is a right triangle whose long side has length 1, and the two short sides have lengths \( \frac{1}{2} \) and \( h \), with \( h \) being the altitude. By the Pythagorean Theorem:
\[
h = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.
\]