Undergraduate Colloquium

Mathematics Undergraduate Colloquium
Wednesday, September 6
12:55 - 1:45
LCB 225

Nicholas Cahill will present

**Bias and Threat: Understanding Sexism in STEM**

**Abstract:** One of the largest ongoing projects in American education has been the attempt to understand the role sexism plays in gaps in performance and achievement between men and women in the sciences. While it may at first have seemed like a simple attitude problem, the gap has proven to be a complicated challenge, with many subtle and less subtle factors at play in the minds of educators and students, and in the broader environment where learning takes place. As the tenacity of this problem has become clear, it is more important now than ever for mathematicians to be understand the threat! This talk will be a brief overview of some of the most important concepts we use to understand what sexism is and how it operates in the sciences.
• beginning with Calculus proper!
• We will now be going more slowly
• but every day will have something new
• and every day will build on what we did the previous day.
• Make sure you stay on top of things!
• Recall the two tasks we set for ourselves:
• Compute the slope of a tangent:
• \( s(t) \) location, \( v(t) \) velocity
• \( v(t) \approx \frac{s(t+h) - s(t)}{h} \)
• Looks like \( \frac{0}{0} \).
• Compute the area underneath the graph of a curve.

• Our approach will be to chop the interval into subintervals, approximate the area in each subinterval by the area of a box, add up the areas of the boxes, and ask what happens as the number of boxes goes to infinity and the size of each box goes to zero.

• This gives rise to something that looks like $0 \times \infty$. 
1.1 Limits

- to deal with indeterminate expressions like $\frac{0}{0}$ and $0 \times \infty$ we need the concept of limits.

- That’s the contents of section 1.1.

- Example 2, page 57.

- What happens to

$$y = \frac{x^2 - x - 6}{x - 3}$$

as $x$ gets close to 3?

- Some numerical values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{x^2 - x - 6}{x - 3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>4.9</td>
</tr>
<tr>
<td>2.99</td>
<td>4.99</td>
</tr>
<tr>
<td>3.01</td>
<td>5.01</td>
</tr>
<tr>
<td>3.1</td>
<td>5.1</td>
</tr>
</tbody>
</table>

- Looks like $y$ gets close to 5.

- We write this as

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} = 5$$

and say “The limit of $\frac{x^2 - x - 6}{x - 3}$ as $x$ approaches 3 equals 5.”

- Numerically approximating a limit often works pretty well!
• We can also do this analytically, as shown here:

\[
\frac{x^2 - x - 6}{x - 3} = \frac{(x+2)(x-3)}{x - 3} = x+2 \quad x \neq 3
\]

\[
\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} = \lim_{x \to 3} (x+2) = 5
\]
Example 3:  
\[ \lim_{x \to 0} \frac{\sin x}{x} = ? \]

- Make sure you set your calculator to radian mode!

- Numerical Values:

\[
\begin{array}{c|c}
  x & \frac{\sin x}{x} \\
  \hline 
  0.1 & 0.998334167 \\
  0.01 & 0.999983333 \\
  0.001 & 0.999999833 \\
\end{array}
\]

- Query: what about negative \( x \)?

- Limit looks like 1.

- It is! Geometric Argument:
Intuitive Meaning of Limit

\[ \lim_{x \to c} f(x) = L \]

means that \( f(x) \) gets arbitrarily close to \( L \) as \( x \) gets sufficiently close to \( c \).

- We can make \( f(x) \) as close to to \( L \) as we wish. All we have to do is pick \( x \) as close to \( c \) as we have to.

- Major point: It does not matter what happens when \( x = c \). \( f(c) \) may be undefined, or it could be some value other than \( L \).

- The limit may be obvious:
  Example: \( \lim_{x \to 3} (4x - 5) = 7 \)

- In this example, nothing goes wrong at \( x = 3 \). But of course we may be in trouble with expressions like \( \frac{0}{0} \) or \( 0 \times \infty \).

- Numerical evaluation does not always tell the limit, and of course it fails when there are parameters in the problem.

- Example: we computed on day 3 of our class:

\[
\lim_{h \to 0} \frac{16(t + h)^2 - 16t^2}{h} = 32t.
\]
• For example, suppose you measure angles in degrees. (Set your calculator to degrees mode.)

• What is

$$\lim_{x \to 0} \frac{\sin x^\circ}{x}$$?

• Numerically, we get

\[
x & \quad \frac{\sin x^\circ}{x} \\
0.1 & \quad 0.017453284 \\
0.01 & \quad 0.017453292 \\
0.001 & \quad 0.017453293
\]

• (Somewhat tricky) exercise: Show that

$$\lim_{z \to 0} \frac{\pi \cdot \sin z}{z} = \lim_{x \to 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$$

\[
\sin x^\circ = \sin \left( \frac{x \cdot \pi}{180} \right) = \left( \sin \left( \frac{x \cdot \pi}{180} \right) \right) \frac{\pi}{180} = \frac{\pi}{180} \sin z
\]

\[
z = \frac{\pi \cdot x}{180}
\]
Examples of Limits that do not exist

\[ \lim_{{x \to 0}} \frac{1}{x^2}. \]

\[ f(x) = \frac{1}{x^2} \]

\[ f(1) = 1 \]
\[ f\left(\frac{1}{10}\right) = 100 \]
\[ f\left(\frac{1}{1000}\right) = 10^6 \]
\[
\lim_{x \to 0} \frac{1}{x}.
\]

\[
f(x) = \frac{1}{x}
\]

\[
f\left(\frac{1}{10}\right) = 10
\]

\[
f\left(\frac{-1}{10}\right) = -10
\]
\[ y = \sin \alpha \]

\[ \lim_{x \to 0} \sin \frac{1}{x} \]
• Graph is impossible to draw.

Figure 1. “Graph” of \( f(x) = \sin \frac{1}{x} \).

from https://i.stack.imgur.com/cOgVv.png

See also Figure 9, page 58, textbook.
One sided Limits

• Example:

\[ \lim_{x \to 0^+} \sqrt{x} = 0. \]

• Greatest Integer Function

\[ [x] = \text{the greatest integer } \leq x. \]

• For example:

\[ [3.1] = 3 \]
\[ [2.9] = 2 \]
\[ [3] = 3 \]
\[ [-3.1] = -4 \]

\[ \lim_{x \to 2^+} [x] = 2 \]
\[ [2.1] = 2 \]
\[ \lim_{x \to 2^-} [x] = 1 \]
\[ [1.9] = 1 \]
Figure 2. Graph of $f(x) = \lfloor x \rfloor$. 

from http://jennarocca.com/greatest-integer-function-equation/
One Sided Limits