Math 1210-23 Notes of 1/19/24

Announcements

- hw 1 due Monday
- hw 2 due Wednesday
- hw 3 open, due next Wednesday

Exam 1 on Chapter 1 next Wednesday. Finish hw 3 before the exam, even though it only closes that evening!

- Study Session today after class, right here

1.4 Limits Involving Trig Functions

- Recall the definition of angles, sine, and cosine

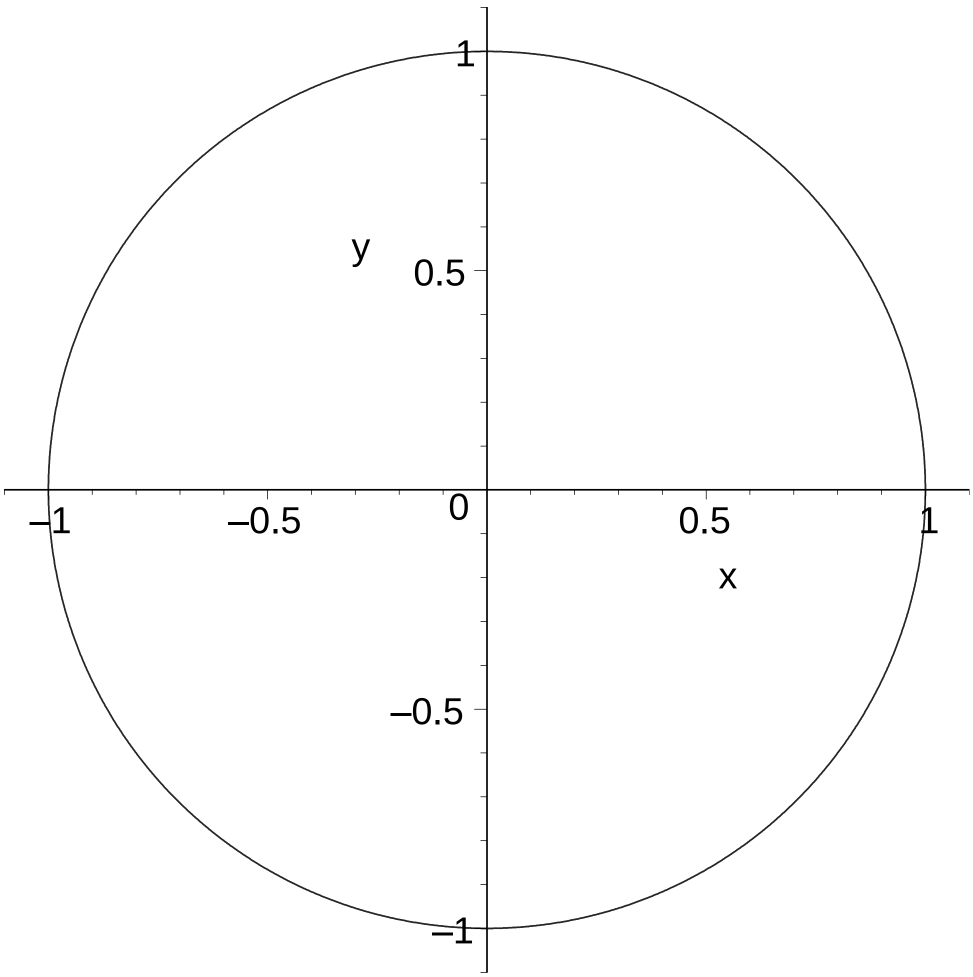

Figure 1. Sine and Cosine.

- Recall

$$\tan t = \frac{\sin t}{\cos t}$$

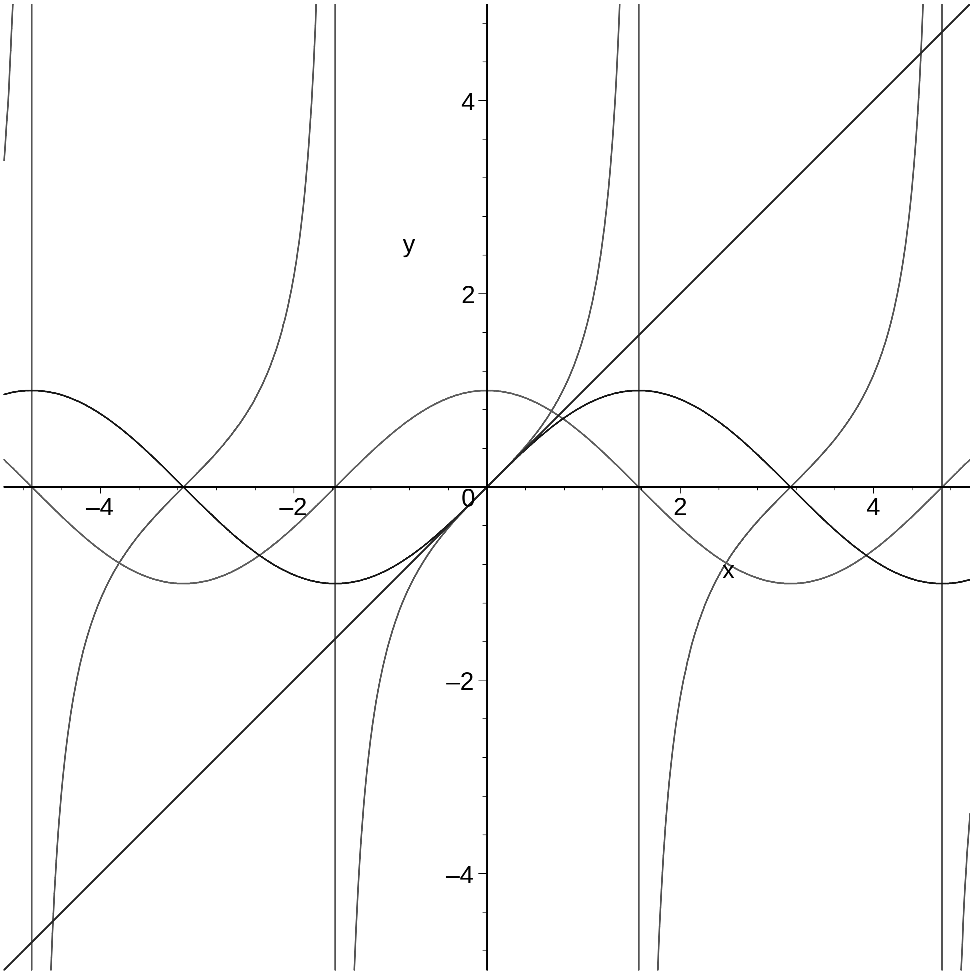

Figure 2. Graphs of sin, cos, tan, x.

- It's clear from the graph that

$$\lim_{t \longrightarrow c} \sin t = \sin c \quad \text{and} \quad \lim_{t \longrightarrow c} \cos t = \cos c$$

- There is a more rigorous discussion (Theorem A, page 74) in the textbook.

- We will need the following limits:

$$\lim_{t\longrightarrow 0}\frac{\sin t}{t}=1 \quad \text{and} \quad \lim_{t\longrightarrow 0}\frac{1-\cos t}{t}=0.$$

- This is the contents of Theorem B on page 75.
- We already saw the first statement:

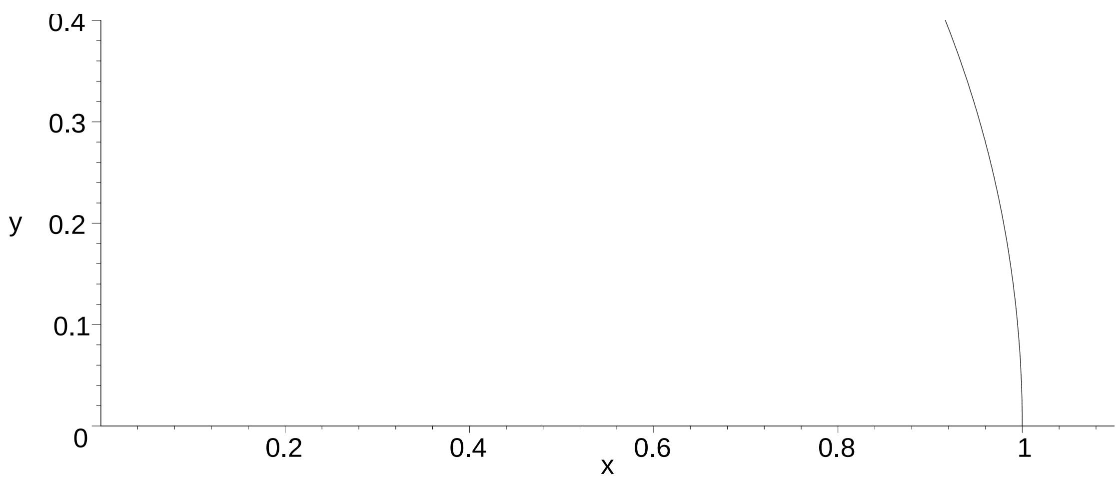

Figure 3. $\displaystyle\lim_{t\longrightarrow 0}\frac{\sin t}{t}=1.$

- Let's look a little closer. Recall

The Squeeze Theorem. Suppose f, g, and h are functions such that

$$f(x)\leq g(x)\leq h(x)$$

for all x near c except possibly *at* $x=c$. Also assume that

$$\lim_{x\longrightarrow c} f(x)=\lim_{x\longrightarrow c} h(x)=L$$

Then

$$\lim g(x)=L$$

- Now recall the area of a sector of a circle:

$$\text{area} = \frac{t}{2} r^2.$$

- Apply to the sector shown here:

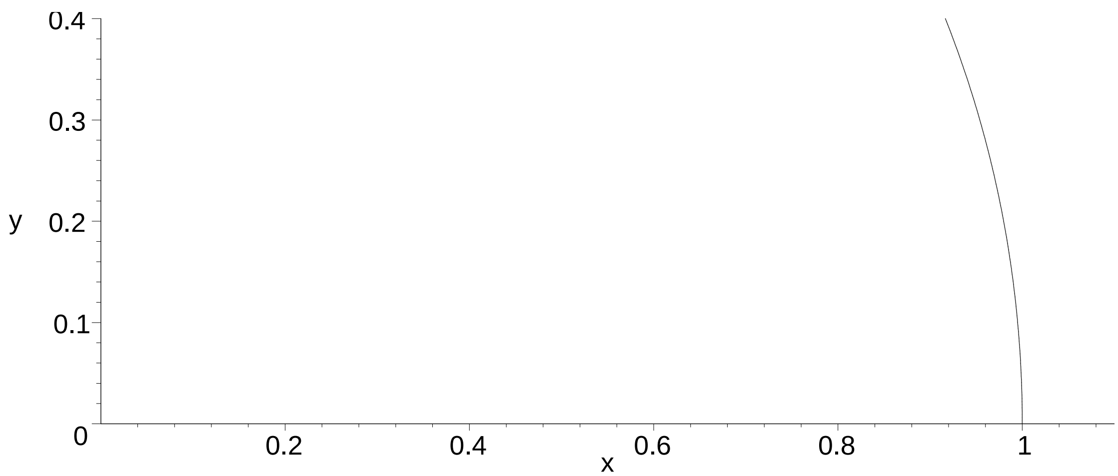

Figure 4. Squeezing the Sine.

$$O = (0,0), \quad A = (1,0), \quad B = (\cos t, 0), \quad P = (\cos t, \sin t), \quad C = (\cos^2 t, \cos t \sin t)$$

and

$$\begin{aligned} f(t) &= \text{area}(\text{sector}OBC) = \frac{t}{2}\cos^2 t \\ g(t) &= \text{area}(\Delta OBP) = \frac{1}{2}\cos t \sin t \\ h(t) &= \text{area}(\text{sector}OAP) = \frac{t}{2}. \end{aligned}$$

- Clearly

$$f(t) \le g(t) \le h(t)$$

or

$$\frac{t}{2}\cos^2 t \le \frac{1}{2}\cos t \sin t \le \frac{t}{2}.$$

Multiply with 2 and divide by $t \cos t$ to get

$$\cos t \leq \frac{\sin t}{t} \leq \frac{1}{\cos t}$$

Clearly

$$\lim_{t \longrightarrow 0} \cos t = \lim_{t \longrightarrow 0} \frac{1}{\cos t} = 1$$

and, by the Squeeze Theorem

$$\lim_{t \longrightarrow 0} \frac{\sin t}{t} = 1.$$

- This implies that

$$\lim_{t \longrightarrow 0} \frac{1 - \cos t}{t} = 0 :$$

More Limits

$$\lim_{\theta \longrightarrow 0} \frac{\tan \theta}{\theta} =$$

$$\lim_{x \longrightarrow 0} \frac{\sin x^\circ}{x} =$$

$$\lim_{\theta \longrightarrow 0} \frac{\tan 5\theta}{\sin 2\theta} =$$

1.6 Continuity

- Remember

  Concept $\longrightarrow$ Definition $\longrightarrow$ Properties $\longrightarrow$ Work

- Intuition: A function is **continuous** if its graph can be drawn without lifting the pencil.

- Continuous or not:

$f(x) = x^2$

$f(x) = \sin x, \cos x$

$f(x) = |x|$

$f(x) = \frac{1}{x}$

$f(x) = \tan x$

$$f(x) = \begin{cases} 0 & \text{if } x = \pi \\ 1 & \text{else} \end{cases}$$

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$