Today is a review of trigonometric functions and polynomials.

**Trigonometric Functions**

- Trigonometry is the mathematics of angles.
- Angles are measured counterclockwise along the unit circle (radius 1 centered at the origin) starting at the $x$-axis.
- The coordinates of a point on the circle are the cosine and sine of the angle, respectively.

\[
2\pi \text{ rad} = 360^\circ \\
\frac{x \text{ rad}}{2\pi} = \frac{360^\circ}{x^\circ} \\
x^\circ = \frac{2\pi}{360} \text{ rad}
\]
• If the angle is between 0 and $\pi/2$ we can compute the trig functions by considering a right triangle.

\[
\sin t = \frac{\text{opp}}{\text{hyp}} \\
\cos t = \frac{\text{adj}}{\text{hyp}} \\
\tan t = \frac{\text{opp}}{\text{adj}} = \frac{\sin t}{\cos t}
\]
Figure 1. Angles. $2\pi = 360^\circ$.

- from http://criselportfolio20122013.weebly.com/trigonometry.html
• The **unit circle** is the circle of radius 1 centered at the origin.

• **Definition** of sine and cosine: If \((x, y)\) is a point on the unit circle, corresponding to an angle \(t\), then

\[
x = \cos t \quad \text{and} \quad y = \sin t
\]

• More notation:

\[
\begin{align*}
\sin t &= \sin(t) \\
\sin^2 t &= (\sin t)^2 \\
\sin^2 t &= \sin(t^2) \\
\cos t &= \cos(t) \\
\cos^2 t &= (\cos t)^2 \\
\cos^2 t &= \cos(t^2)
\end{align*}
\]

• For example \(\sin t + c\) is ambiguous and should be written as \(\sin(t + c)\) or \(c + \sin t\).

• Some immediate consequences of the definition:

\[
\begin{align*}
\sin^2 t + \cos^2 t &= 1 \\
\text{because on the unit circle } x^2 + y^2 &= 1 \\
\sin(t + 2\pi) &= \sin t \\
\cos(t + 2\pi) &= \cos t \\
2 \pi \text{ periodicity}
\end{align*}
\]

• Less obvious consequences

\[
\begin{align*}
\cos t &= \sin \left( t + \frac{\pi}{2} \right) \\
\sin t &= \cos \left( t - \frac{\pi}{2} \right) \\
\text{sin lags cos by } \frac{\pi}{2}
\end{align*}
\]
• 4 more trigonometric functions are defined by

\[
\tan t = \frac{\sin t}{\cos t} \quad \text{tangent}
\]

\[
\cot t = \frac{\cos t}{\sin t} \quad \text{cotangent}
\]

\[
\sec t = \frac{1}{\cos t} \quad \text{secant}
\]

\[
\csc t = \frac{1}{\sin t} \quad \text{cosecant}
\]

• We usually only use sine, cosine, and tangent.

**Figure 2.** Graphs of sin, cos, tan, x.

• Note that the tangent function is \(\pi\)-periodic (not just \(2\pi\)-periodic)

• Also note that the functions \(x\), \(\sin x\), and \(\tan x\) are all tangent at the origin. This is no coincidence!
Trigonometric Identities

- There is a host of them. We’ll use some of them occasionally in this class.
- Here is a partial list:

\[
\begin{align*}
\sin(x + y) &= \sin x \cos y + \cos x \sin y \\
\cos(x + y) &= \cos x \cos y - \sin x \sin y \\
\tan(x + y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\
\sin 2x &= 2\sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
\sin \left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos x}{2}} \\
\cos \left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 + \cos x}{2}}
\end{align*}
\]

Do not attempt to memorize those formulas!

On the other hand, it should be obvious that

\[\sin^2 x + \cos^2 x = 1.\]
“Inverse” Trig Functions

• major fact: Trig functions are not invertible.

• This is because for any value of a trigonometric function there are infinitely many angles that give rise to that value of the trig function.

• The trig functions fail the horizontal line test because of their periodicity.

• However, one can define functions such that

\[
\begin{align*}
\sin \arcsin x &= x & -1 \leq x \leq 1 & -\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2} & \text{(arcsine)} \\
\cos \arccos x &= x & -1 \leq x \leq 1 & 0 \leq \arccos x \leq \pi & \text{(arccosine)} \\
\tan \arctan x &= x & -\infty < x < \infty & -\frac{\pi}{2} < \arctan x < \frac{\pi}{2} & \text{(arctangent)}
\end{align*}
\]

• But note that in general

\[
\begin{align*}
\arcsin \sin x \neq x \\
\arccos \cos x \neq x \\
\arctan \tan x \neq x
\end{align*}
\]

• For example:

\[
\begin{align*}
\arcsin \sin(2\pi) &= 0 \\
\arccos \cos(-\pi) &= \pi \\
\arctan \tan 2.0 &\approx -1.141592
\end{align*}
\]
Polynomials

- A function $P$ is a polynomial (in $x$) if it can be written as

$$P(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0.$$

- The $a_i$ are the coefficients of the polynomial.
- If the leading coefficient ($a_n \neq 0$) then $n$ is the degree of the polynomial.

$$3x^3 + 4x^4 - 5$$

$$(x+1)(x^2 + r)(x^3 + r)$$

$7 \quad 3 \quad 3 \quad 2 \quad 6 \quad \text{majority}$$
More Vocabulary

- Low degrees have their own names:

<table>
<thead>
<tr>
<th>n</th>
<th>Name</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>constant</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>linear</td>
<td>$4x - 5$</td>
</tr>
<tr>
<td>2</td>
<td>quadratic</td>
<td>$x^2 + 1$</td>
</tr>
<tr>
<td>3</td>
<td>cubic</td>
<td>$x^3 - 2x + 1$</td>
</tr>
<tr>
<td>4</td>
<td>quartic</td>
<td>$14x^4$</td>
</tr>
<tr>
<td>5</td>
<td>quintic</td>
<td>$17x^5$</td>
</tr>
</tbody>
</table>

- The list goes on: sextic, septic, octic, nonic

- monomial: one term only.

- binomial: two terms

- trinomial: three terms

- constant term: $a_0$

- leading term: $a_n x^n$

- leading coefficient: $a_n$
Example

\[ p(x) = 2x^4 + 3x^3 - 4x^2 - 5x + 1 \]
\[ = (2x^3 + 3x^2 - 4x - 5)x + 1 \]
\[ = ((2x^2 + 3x - 4)x - 5)x + 1 \]
\[ = (((2x + 3)x - 4)x - 5)x + 1 \]

\[ p(2) = 2 \cdot 2^4 + 3 \cdot 8 - 4 \cdot 4 - 5 \cdot 2 + 1 \]
\[ = 32 + 24 - 16 - 10 + 1 \]
\[ = \boxed{31} \]

but also:

\[ p(2) = (((((2 \cdot 2 + 3) - 4) \cdot 2 - 5)x + 1 \]
\[ = 4 \quad 7 \quad 14 \quad 10 \quad 20 \quad 15 \quad 30 \quad \boxed{31} \]