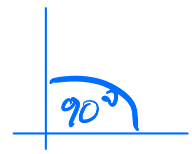
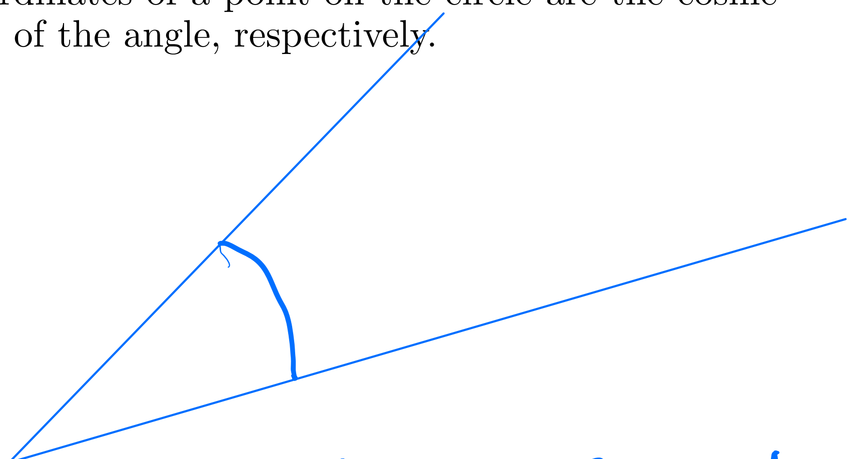
Math 1210-23 **Notes of 1/17/24**

- Today is a review of trigonometric functions and polynomials.

Trigonometric Functions

- Trigonometry is the mathematics of angles.
- angles are measured counterclockwise along the unit circle (radius 1 centered at the origin) starting at the x-axis.
- The coordinates of a point on the circle are the cosine and sine of the angle, respectively.

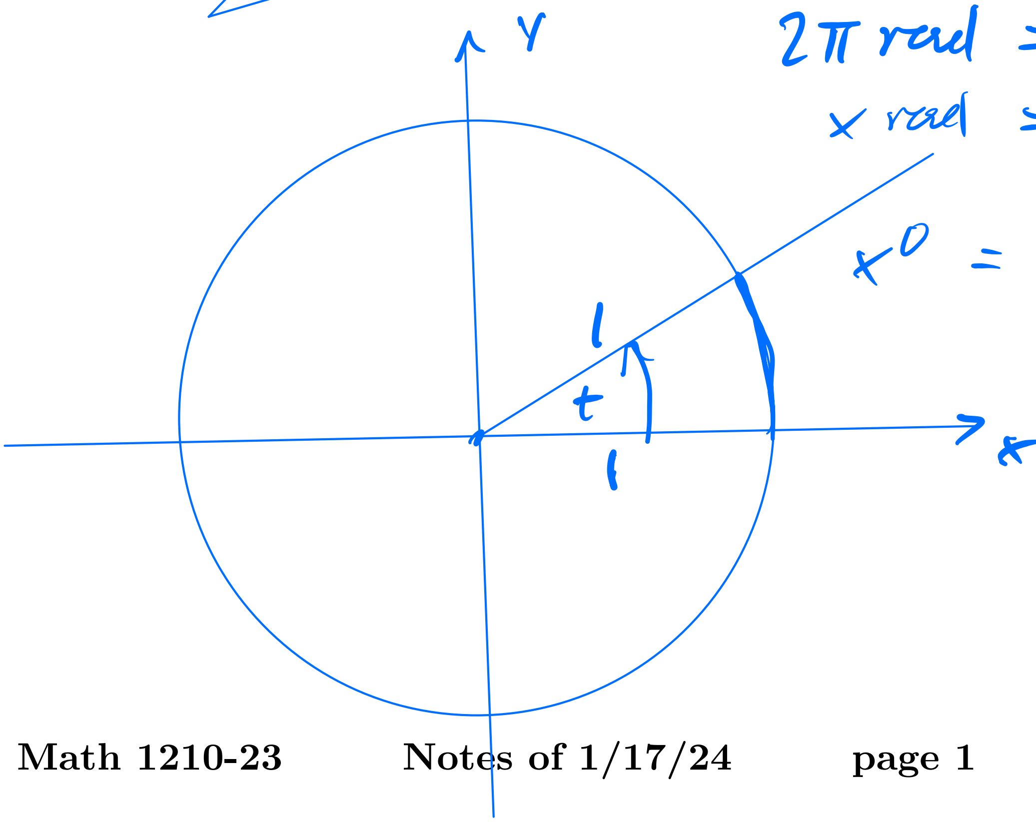




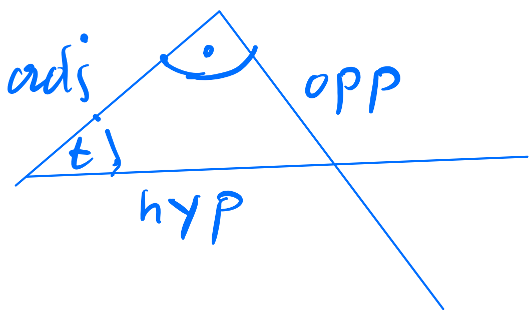
$$2\pi \text{ rad} = 360^\circ$$

$$x \text{ rad} = \frac{360}{2\pi} x^\circ$$

$$x^\circ = \frac{2\pi}{360} \text{ rad}$$



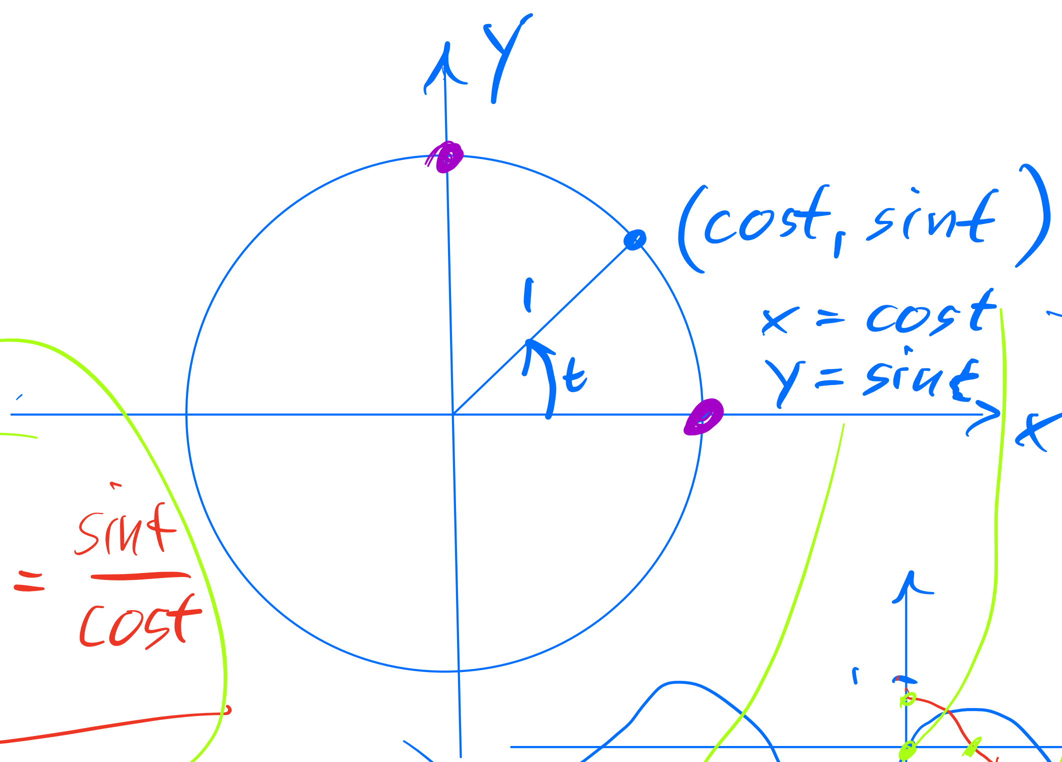
- If the angle is between 0 and $\pi/2$ we can compute the trig functions by considering a right triangle.



$$\sin t = \frac{opp}{hyp}$$

$$\cos t = \frac{adj}{hyp}$$

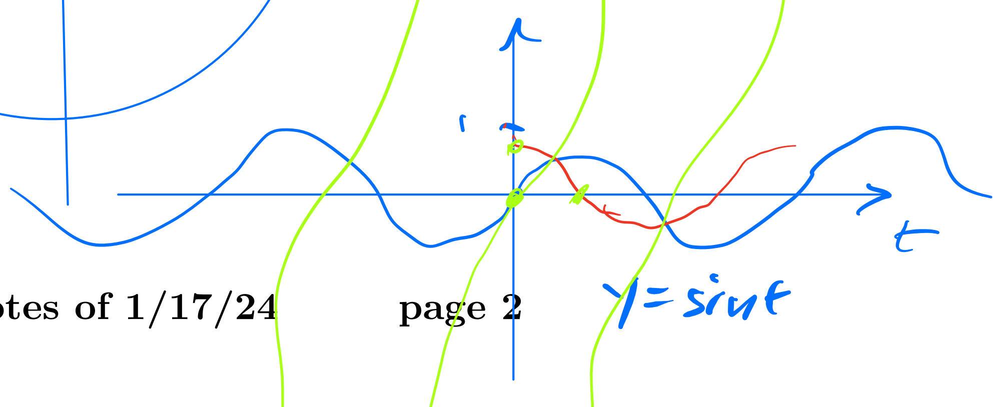
$$\tan t = \frac{opp}{adj} = \frac{\sin t}{\cos t}$$



$(\cos t, \sin t) = (x, y)$

$x = \cos t$
$y = \sin t$

$$\tan t = \frac{\sin t}{\cos t}$$



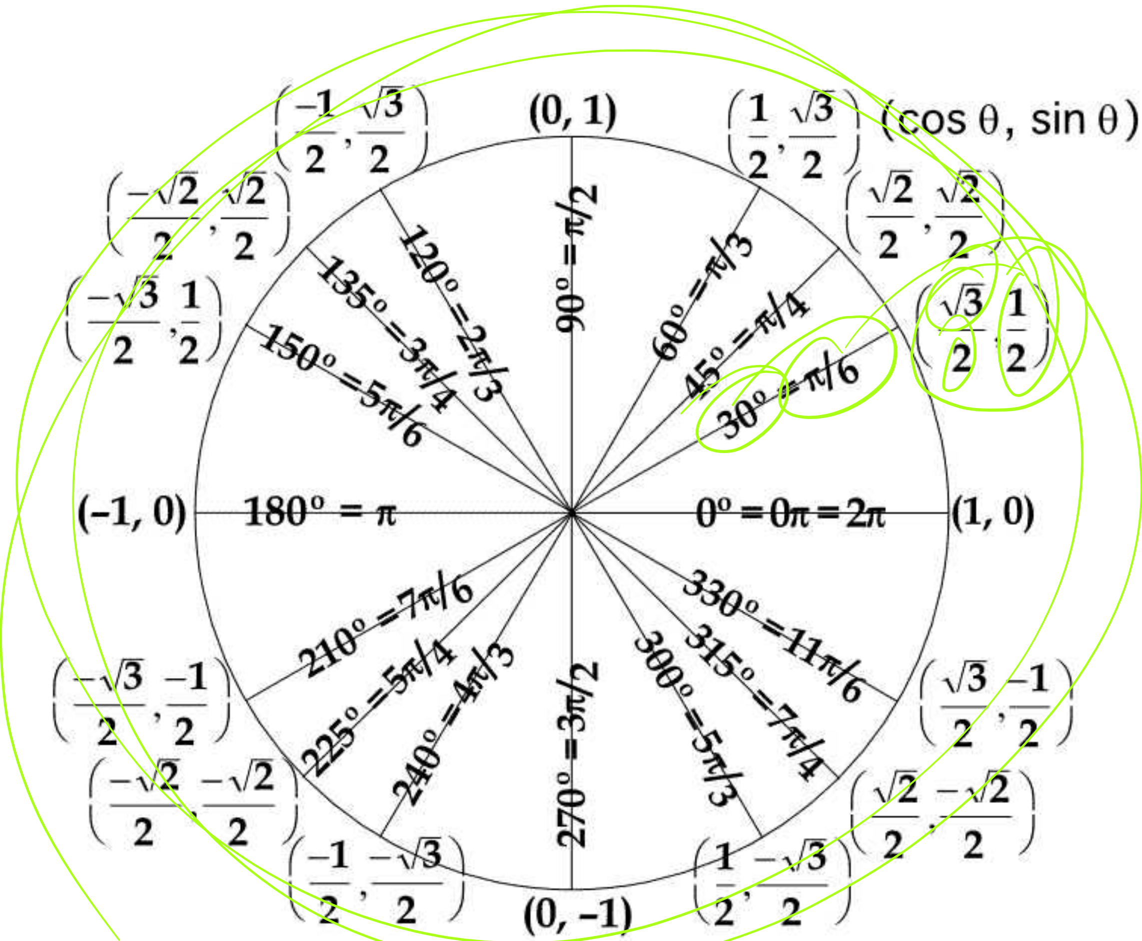


Figure 1. Angles. $2\pi = 360°$.

- from `http://criselportfolio20122013.weebly.com/trigonometry.html`

- The **unit circle** is the circle of radius 1 centered at the origin.
- **Definition** of sine and cosine: If (x, y) is a point on the unit circle, corresponding to an angle t, then

$$x = \cos t \quad \text{and} \quad y = \sin t$$

sin(t) = sin t

- More notation:

$$\begin{aligned} \sin t &= \sin(t) \\ \sin^2 t &= (\sin t)^2 \\ \sin t^2 &= \sin(t^2) \\ \cos t &= \cos(t) \\ \cos^2 t &= (\cos t)^2 \\ \cos t^2 &= \cos(t^2) \end{aligned}$$

(sin(t))^2

- For example $\sin t + c$ is ambiguous and should be written as $\sin(t + c)$ or $c + \sin t$.

(sin t) +c

- Some immediate consequences of the definition:

$$\sin^2 t + \cos^2 t = 1$$

because on the unit circle $x^2 + y^2 = 1$

$$\sin(t + 2\pi) = \sin t$$

$$\cos(t + 2\pi) = \cos t$$

$2\ \pi$ periodicity

- Less obvious consequences

$$\cos t = \sin\left(t + \frac{\pi}{2}\right)$$

$$\sin t = \cos\left(t - \frac{\pi}{2}\right)$$

sin lags cos by $\frac{\pi}{2}$

- 4 more trigonometric functions are defined by

$$\begin{aligned}
\tan t &= \frac{\sin t}{\cos t} && \text{tangent} \\
\cot t &= \frac{\cos t}{\sin t} && \text{cotangent} \\
\sec t &= \frac{1}{\cos t} && \text{secant} \\
\csc t &= \frac{1}{\sin t} && \text{cosecant}
\end{aligned}$$

- We usually only use sine, cosine, and tangent.

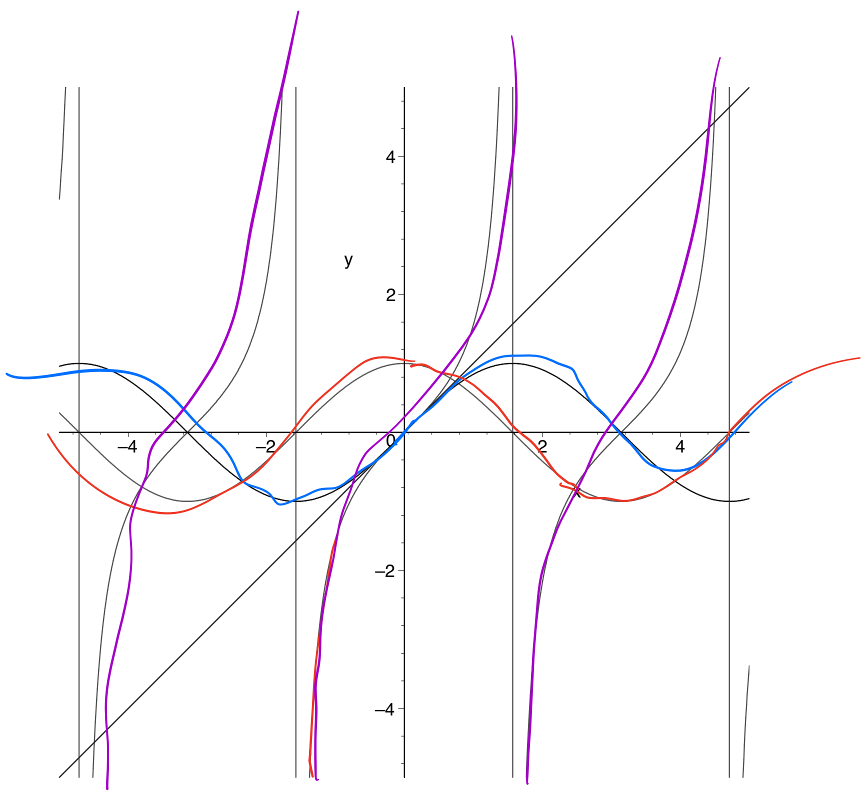


Figure 2. Graphs of sin, cos, tan, x.

- Note that the tangent function is π-periodic (not just 2π-periodic)
- Also note that the functions x, $\sin x$, and $\tan x$ are all tangent at the origin. This is no coincidence!

Trigonometric Identities

- There is a host of them. We'll use some of them occasionally in this class.
- Here is a partial list:

$$
\begin{aligned}
\sin(x+y) &= \sin x \cos y + \cos x \sin y \\
\cos(x+y) &= \cos x \cos y - \sin x \sin y \\
\tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\
\sin 2x &= 2 \sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
\sin\left(\frac{x}{2}\right) &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos\left(\frac{x}{2}\right) &= \pm\sqrt{\frac{1+\cos x}{2}}
\end{aligned}
$$

Do not attempt to memorize those formulas!

On the other hand, it should be obvious that

$$\boxed{\sin^2 x + \cos^2 x = 1.}$$

$f(g(x)) = x$

$\sin^{-1}$
arcsin
asin
$\arcsin(\sin x) = x$

"Inverse" Trig Functions

- major fact: Trig functions are not invertible.
- This is because for any value of a trigonometric function there are infinitely many angles that give rise to that value of the trig function.
- The trig functions fail the horizontal line test because of their periodicity.
- However, one can define functions such that

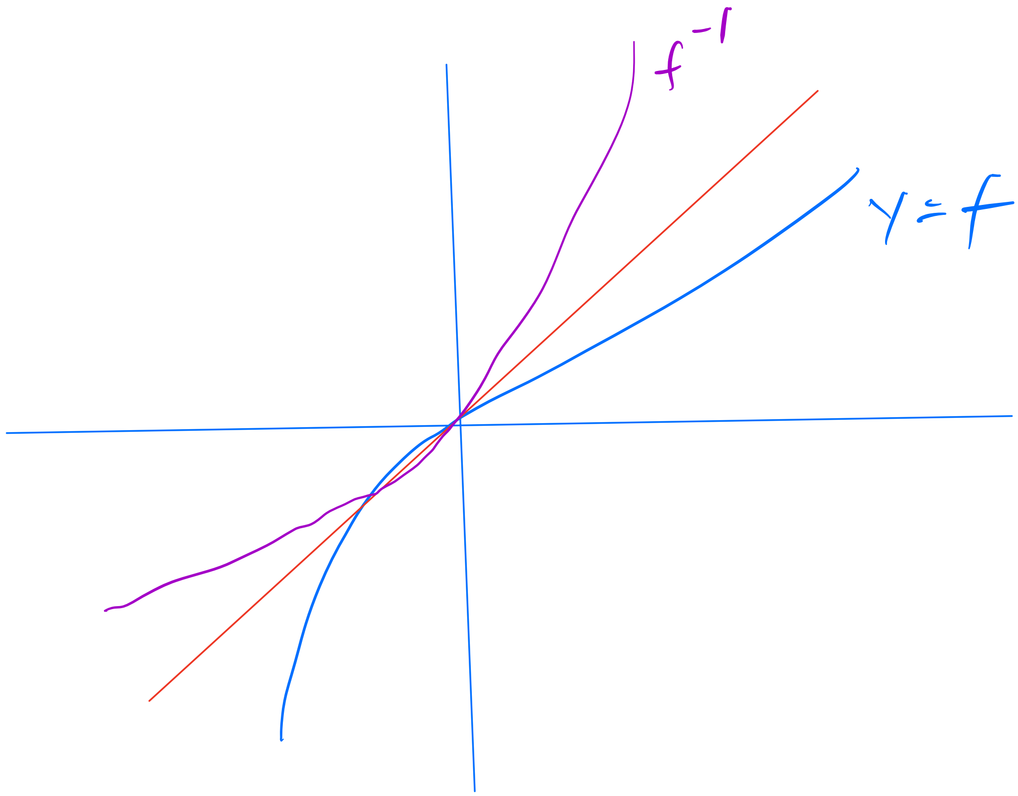
$$
\begin{array}{rclccc}
\sin \arcsin x & = & x & -1 \le x \le 1 & -\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2} & \text{(arcsine)} \\
\cos \arccos x & = & x & -1 \le x \le 1 & 0 \le \arccos x \le \pi & \text{(arccosine)} \\
\tan \arctan x & = & x & -\infty < x < \infty & -\frac{\pi}{2} < \arctan x < \frac{\pi}{2} & \text{(arctangent)}
\end{array}
$$

- But note that in general

$$\arcsin \sin x \ne x$$
$$\arccos \cos x \ne x$$
$$\arctan \tan x \ne x$$

- For example:

$$\arcsin \sin(2\pi) = 0$$
$$\arccos \cos(-\pi) = \pi$$
$$\arctan \tan 2.0 \approx -1.141592$$


f^{-1}
$y = f$

Polynomials

- A function P is a polynomial (in x) if it can be written as

$$P(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0.$$

- The a_i are the **coefficients of the polynomial**.
- If the **leading coefficient** ($a_n \neq 0$) then n is the **degree** of the polynomial.

$3x^3 + 4x^4 - 5$

$(x^2+1)(x^3+1)(x+1)$

7 3

3 2

6 majority

More Vocabulary

- Low degrees have their own names:

n	**Name**	**Example**
0	**constant**	3
1	**linear**	$4x - 5$
2	**quadratic**	$x^2 + 1$
3	**cubic**	$x^3 - 2x + 1$
4	**quartic**	$14x^4$
5	**quintic**	$17x^5$

- The list goes on: sextic, septic, octic, nonic
- **monomial:** one term only. x^5
- **binomial:** two terms $x^3 + x$
- **trinomial:** three terms
- **constant term:** a_0
- **leading term:** $a_n x^n$
- **leading coefficient:** a_n

Long Division

Example

$$p(x) = 2x^4 + 3x^3 - 4x^2 - 5x + 1$$
$$= (2x^3 + 3x^2 - 4x - 5)x + 1$$
$$= ((2x^2 + 3x - 4)x - 5)x + 1$$
$$= (((2x + 3)x - 4)x - 5)x + 1$$

$$p(2) = 2 \cdot 2^4 + 3 \cdot 8 - 4 \cdot 4 - 5 \cdot 2 + 1$$
$$= 32 + 24 - 16 - 10 + 1$$
$$= 31$$

but also:

$$p(2) = (((2 \cdot 2 + 3) \cdot 2 - 4) \cdot 2 - 5)x + 1$$
$$= \quad 4 \quad 7 \quad 14 \quad 10 \quad 20 \quad 15 \quad 30 \quad 31$$