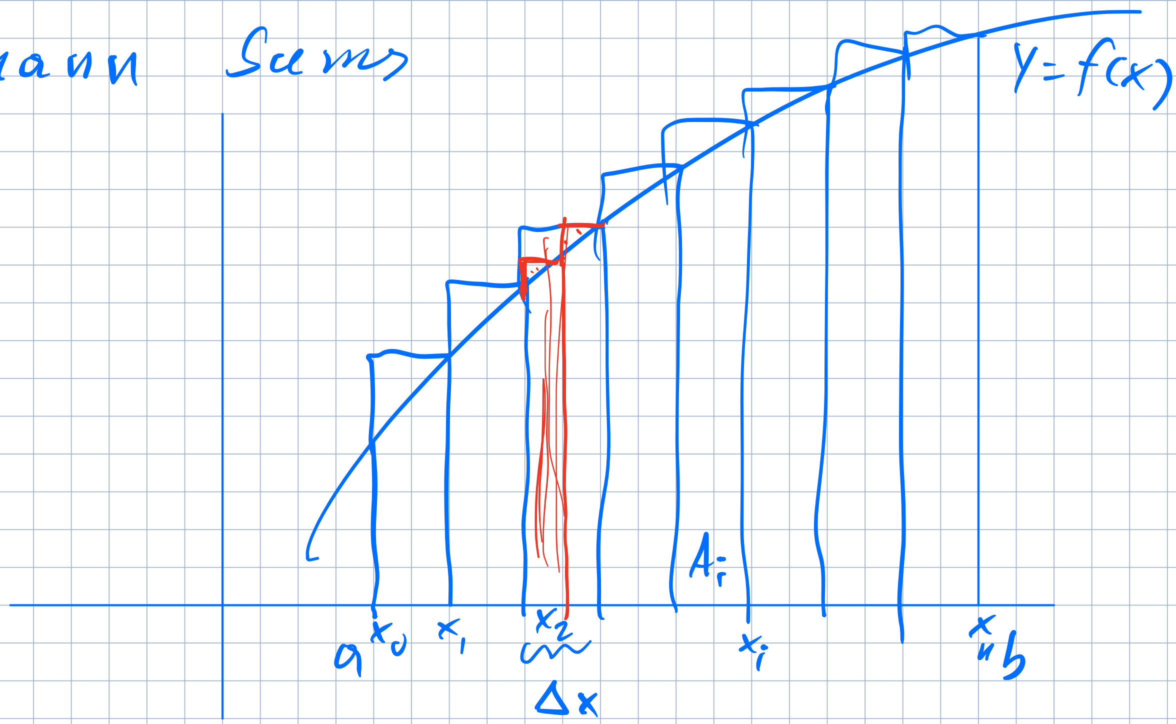
Math 1210-23 4/26/24 Review

Riemann Sums



$$x_i = a + i\Delta x$$

$$a = x_0$$
$$b = x_n$$

$$A_i = f(x_i)\Delta x$$

$$A \approx \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} f(x_i)\Delta x$$

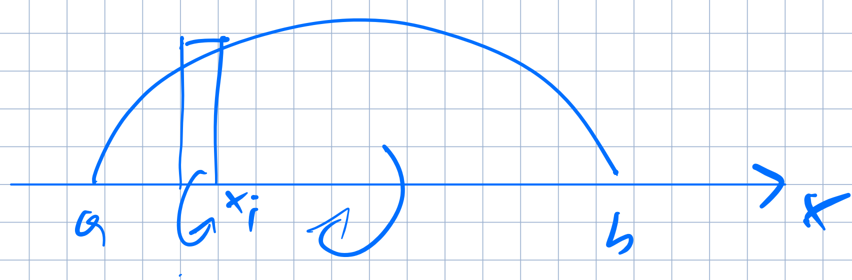
$$A = \int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x =$$

$$\Delta x = \frac{b-a}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\int_0^1 x^2\,dx = \frac{1}{3}$$



$$\Delta V_i = \pi f^2(x_i)\,\Delta x$$

$$V = \int_a^b \pi f^2(x)\,dx$$

$$\Delta x = \frac{2}{n}$$

$$x_i = 1 + i\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin x_i^2\,\Delta x = \int_1^3 \sin x^2\,dx$$

$$a = x_0 = 1 + 0\Delta x = 1$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$F' = f$$

$x_i = i\Delta x$ $\qquad \Delta x = \frac{\pi}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin x_i \,\Delta x = \int_0^\pi \sin x \, dx$$

$$= \left[-\cos x\right]_0^\pi$$

$$= -\cos\pi - (-\cos 0) = -(-1) - (-1)$$

$$= 1 + 1 = 2 \qquad = 2$$

$a = x_0 = 0$

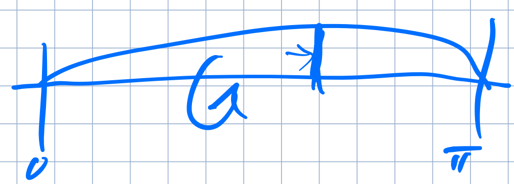
$b = \pi = x_n$

$\cos\pi = -1$

$\cos 0 = 1$

Volume →

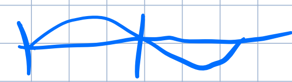
$y = \sin x$



$$V = \int_0^\pi \pi \sin^2 dx =$$

$$\frac{d}{dx}\ \frac{\sin^3 x}{3\cos x} = \frac{3\sin^2 x\cos x\cdot 3\cos x + \sin^3 x\cos x}{9\cos^2 x}$$

$$\int_0^{\pi}\sin^2 x\,d = \frac{1}{2}\int_0^{2\pi}\sin^2 x\,dx$$



$$\int_0^{2\pi}\sin^2 x\,dx = \int_0^{2\pi}\cos^2 x\,dx = \frac{1}{2}\int_0^{2\pi}1\,dx = \frac{1}{2}\cdot 2\pi = \pi$$

$$= \int_0^{2\pi}\sin^2 x + \cos^2 x\,dx = \int_0^{2\pi}1\,dx = 2\pi$$

$$= \int_0^{2\pi}\sin^2 x\,dx + \int_0^{2\pi}\cos^2 x\,dx = 2\pi$$

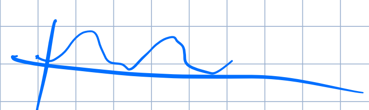
$$=$$

$$= 2\int_0^{2\pi}\sin^2 x\,dx = 2\pi$$

$$\int_0^{2\pi}\sin^2 x\,dx = \pi$$

$$\sin^2 x\,dx = \frac{1}{2}\left(-\cos 2x + 1\right) \quad ?$$

$$= \frac{1}{2}\left(1 - \cos 2x\right)$$



$$\int_0^{2\pi}\sin^2 x\,dx = \frac{1}{2}\int_0^{2\pi}1 - \cos 2x\,dx$$

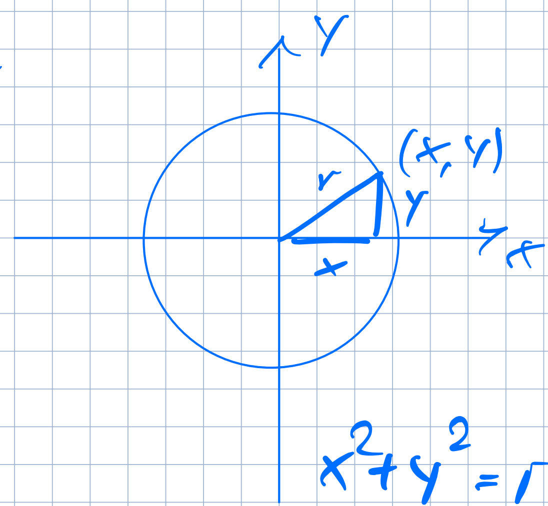
$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\Big|_0^{2\pi} + C_1$$

$$= \frac{1}{2}(2\pi - 0) = \pi$$

$$0 \neq \int_{-3}^{3}\sqrt{9-x^2}\,dx = \frac{9\pi}{2}$$

$$\int_{-r}^{r}\sqrt{r^2-x^2}\,dx = \frac{\pi r^2}{2}$$

$$x^2+y^2=r^2$$



$$x^2+y^2=r^2$$

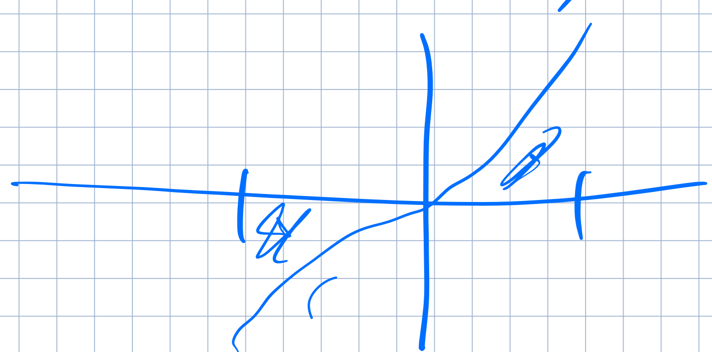
$$y=\sqrt{r^2-x^2}$$

$$\int_{-\sqrt{\pi}}^{\sqrt{\pi}} \sin x^3\,dx = 0$$

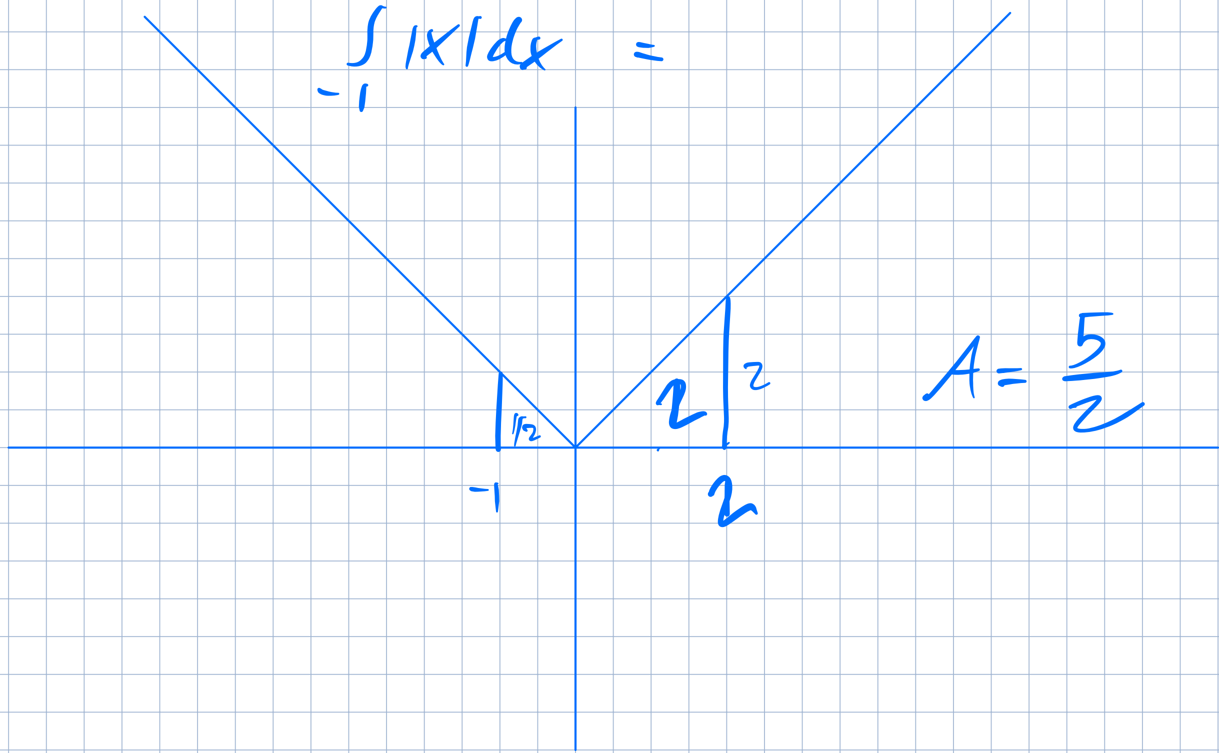
$$\sin(-z) = -\sin z$$

$$f(-x) = -f(x)$$

$$\sin(-x)^3 = \sin(-x^3) = -\sin x^3$$



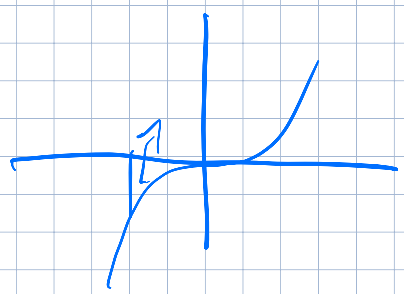
$$\int_{-1}^{2} |x|\,dx =$$



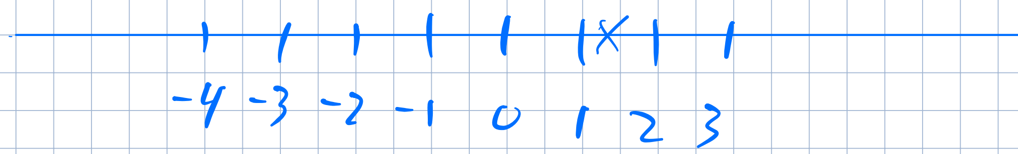
$$A = \frac{5}{2}$$

$$\int_{-1}^{2} |x^3|\,dx = \int_{-1}^{0} -x^3\,dx + \int_{0}^{2} x^3\,dx = -\frac{x^4}{4}\Big|_{-1}^{0} + \frac{x^4}{4}\Big|_{0}^{2}$$

$$= \frac{1}{4} + 4 = \frac{17}{4}$$

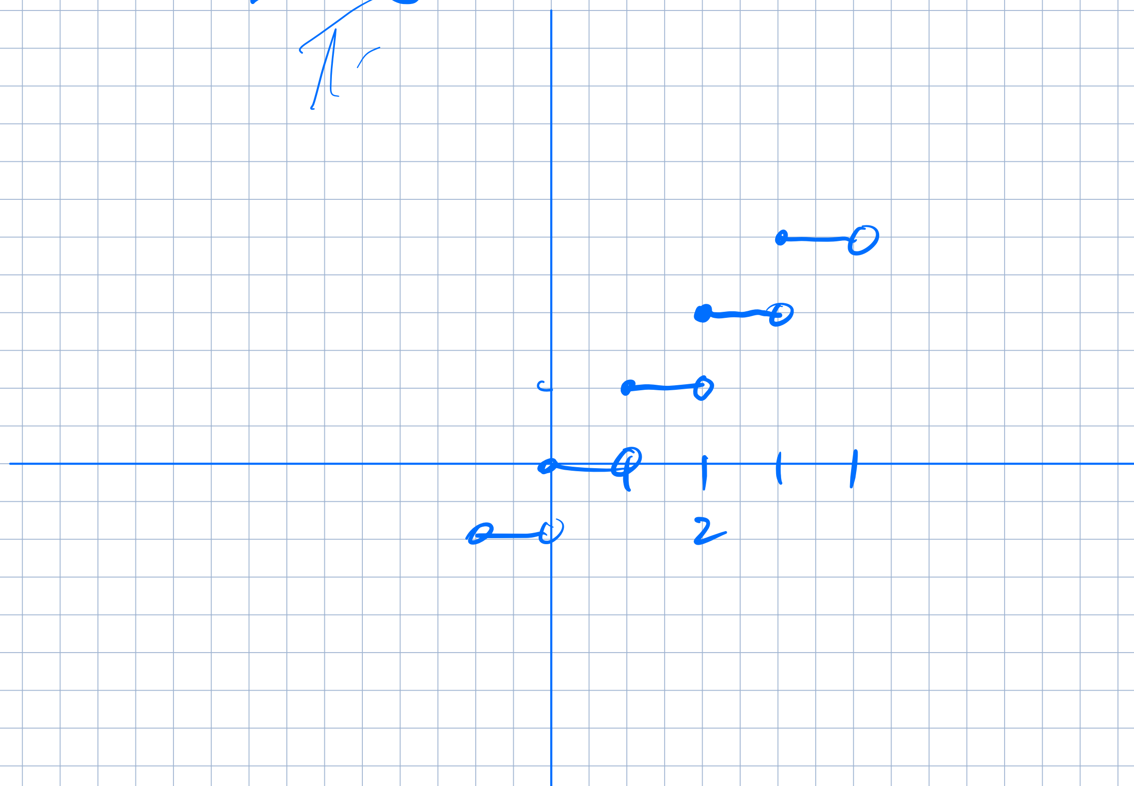


$[\![x]\!]$ = greatest integer $\leq x$



$[\![1.1]\!] = 1$ $\qquad$ $[\![1]\!] = 1$

$-1 \neq [\![-1.1]\!] = -2$

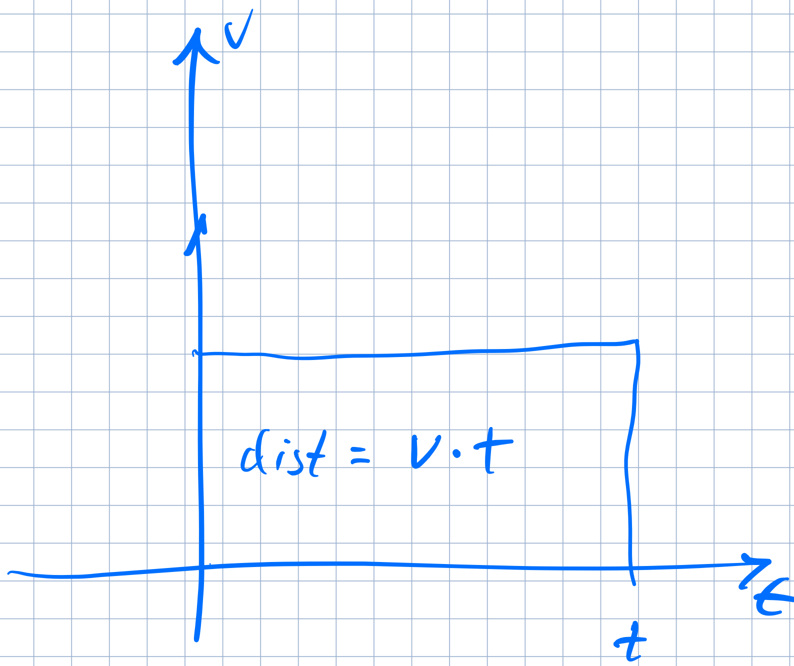


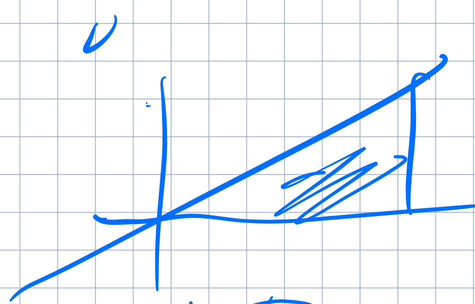
$$\lim_{x \to 1^-} [\![x]\!] = 0$$

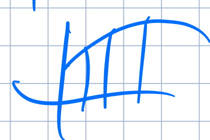
$$\lim_{x \to 1^+} [\![x]\!] = 1 = [\![1]\!]$$

$\lim_{x \to c} f(x) = f(c)$ cont. at c

Work







$L =$ natural length

Hooke's Law

$F = k \cdot x$ $\qquad$ x extension

k spring constant $\qquad$ Length − natural Length

$$k = \frac{F}{x}$$

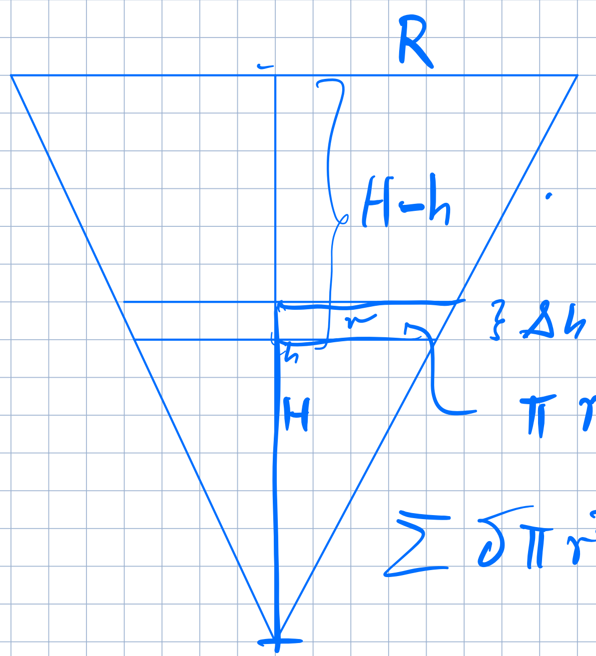
It takes $F = 2\,lb$ to extend a spring by $\frac{1}{4}$ foot

how much work to extend it by 2 feet?

$$k = \frac{2}{1/4} = 8$$

$$F = 8x$$

$$W = \int_0^2 8x\,dx = 4x^2\Big|_0^2 = 16 \text{ ft lbs}$$



δ

$\pi r^2 \Delta h = V$

$\frac{r}{h} = \frac{R}{H}$

$r = \frac{R}{H} h$

$\sum \delta \pi r^2 \Delta h (H-h)$

$$W = \int_0^H \delta \pi r^2 (H-h)\, dh$$

$$= \int_0^H \delta \pi \frac{R^2}{H^2} h^2 (H-h)\, dh$$

$$= \delta \pi \frac{R^2}{H^2} \int_0^H h^2 (H-h)\, dh$$

$$= \delta \pi \frac{R^2}{H^2} \int_0^H h^2 H - h^3\, dh$$

$$= \delta \pi \frac{R^2}{H^2} \left[\frac{h^3 H}{3} - \frac{h^4}{4} \right]_0^H$$

$$= \delta \pi \frac{R^2 H^4}{H^2} \left[\frac{1}{3} - \frac{1}{4} \right]$$

$$W = \frac{1}{12}\,\delta \pi R^2 H^2 \qquad \delta = \frac{lb}{ft^3}$$

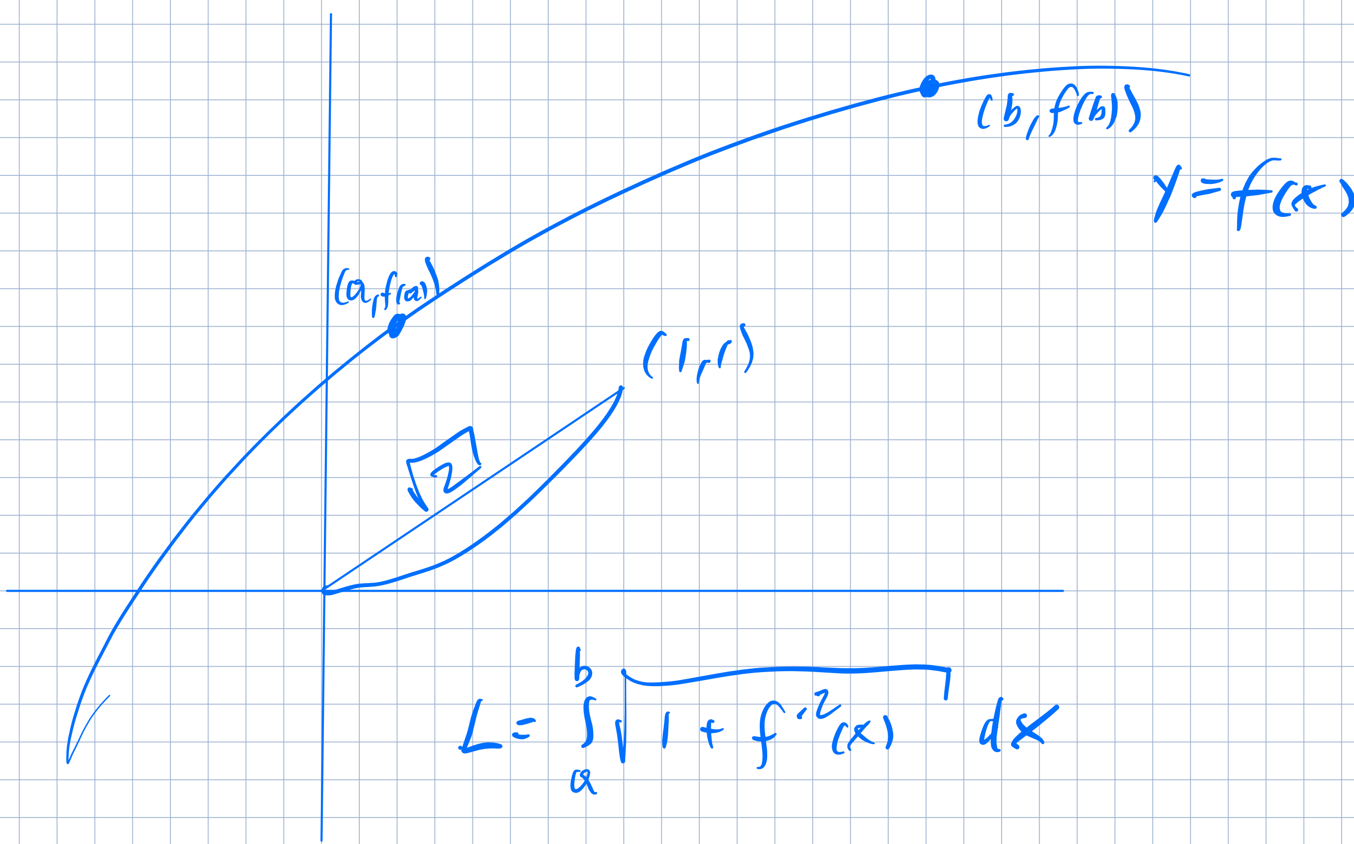
$$W = ft\,lb$$

$$R = ft$$

$$H = ft$$

$$lbf = \frac{\cancel{\pi}}{12}\,\frac{lb\ ft^2 ft^2}{ft^3}$$

$$= lb\,ft$$



$$L = \int_a^b \sqrt{1 + f'^2(x)}\,dx$$

$$f'(x) = x^{1/2} \cdot \frac{3}{2}$$

$$f(x) = x^{3/2}$$

$$[a, b] = [0, 1]$$

$$f'(x) = \frac{3}{2} x^{1/2}$$

$$\left(f'(x)\right)^2 = \frac{9}{4} x$$

$$\int_0^1 \sqrt{1+\frac{9}{4}x}\, dx = \int_0^1 \left(1+\frac{9}{4}x\right)^{1/2} dx$$

$$= \frac{4}{9}\,\frac{2}{3}\left(1+\frac{9}{4}x\right)^{3/2} \Big|_0^1$$

$$= \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1^{3/2}\right)$$

$$I = \int_0^1 \sqrt{1+\frac{9}{4}x}\, dx = \int_1^{13/4} \sqrt{u} \cdot \frac{4}{9} du = \int_1^{13/4} u^{1/2} \cdot \frac{4}{9} du$$

$$= \frac{4}{9} \cdot \frac{2}{3} u^{3/2} \Big|_1^{13/4} = \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1^{3/2}\right)$$

$x \in 0, 1$

$\in 1, \frac{13}{4}$

$$u = 1 + \frac{9}{4} x$$

$$\frac{du}{dx} = \frac{9}{4} \qquad du = \frac{9}{4} dx \qquad dx = \frac{4}{9} du$$

FToC

$$\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_a^x t^t\,dt = x^x$$

$$(x^2)^{x^2} \neq \frac{d}{dx}\int_a^{x^2} t^t\,dt = (x^2)^{x^2}\cdot 2x \qquad F' = f$$

$$\frac{d}{dx}\int_{L(x)}^{u(x)} f(t)\,dt = \frac{d}{dx}\left[F(u(x)) - F(L(x))\right]$$

$$= F'(u(x))\,u'(x) - F'(L(x))\,L'(x)$$

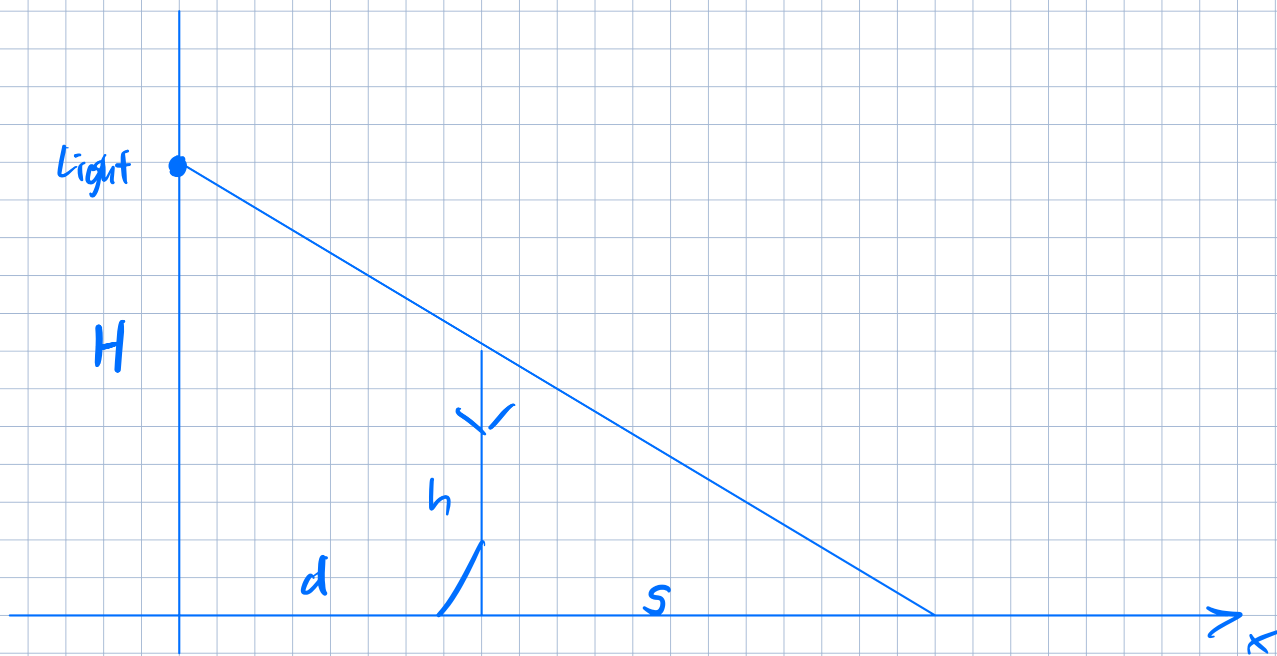
$$= f(u(x))\,u'(x) - f(L(x))\,\underline{L'(x)}$$

$$L(x) = a \qquad L'(x) = 0$$

$$u(x) = x \qquad u'(x) = 1$$

$$= f(x)\cdot 1 - f(a)\cdot 0$$

$$= f(x)$$



$$d' = V$$

$$\frac{d+s}{H} = \frac{s}{h} \qquad ' = \frac{d}{dt}$$

$$\frac{V+s'}{H} = \frac{d'+s'}{H} = \frac{s'}{h}$$

$$\frac{V}{H} = \frac{s'}{h} - \frac{s'}{H} = s'\left(\frac{1}{h} - \frac{1}{H}\right)$$

$$s' = \frac{V \qquad hH}{H\left(\frac{1}{h} - \frac{1}{H}\right) hH} = \frac{VhH}{H(H-h)} = \frac{Vh}{H-h}$$

$$\text{Answer} = V + \frac{Vh}{H-h} = V\left(1 + \frac{h}{H-h}\right)$$

$$= V\left(\frac{H-h+h}{H-h}\right) = \frac{VH}{H-h}$$