Review Th 4/25/24

Driving at

$25\ ft/s^2$ 5sec.

$a = -25$

$v(t) = v = -25t + C_1$

$v(5) = -125 + C_1 = 0$

$C = 125$

$v(t) = -25t + 125$

$s(t) = -25\frac{t^2}{2} + 125t + C_1$

$= -25\frac{t^2}{2} + 125t$

$s(5) = -25 \cdot \frac{25}{2} + 125 \cdot 5$

$= -\frac{625}{2} + 625 = \frac{625}{2}$

a acc.

v velocity

s location

$a = s'' = v'$

$v = s'$

$s(0) = 0$

$C_1 = 0$

IBS

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\int f'(g(x))\, \underbrace{g'(x)\, dx}_{du} = f(g(x)) + C$$

$u = g(x)$ $\quad \frac{du}{dx} = g'(x)$ $\quad du = g'(x)dx$

$$= \int f(u)\, du$$

$$I = \int_1^2 2x \sin x^2\, dx = -\cos x^2 \Big|_1^2 = -\cos 4 + \cos 1$$

$u = x^2$ $\quad \frac{du}{dx} = 2x$ $\quad du = 2x\, dx$

$$I = \int \sin u\, du = -\cos u = -\cos x^2 \Big|_{1=x}^{2=x}$$

$$I = \int_{u=1}^{u=4} \sin u\, du = -\cos u \Big|_{1=u}^{4=u} = -\cos x^2 \Big|_{1=x}^{2=x}$$

$$= -\cos 4 + \cos 1$$

$$I = \int_{x=1}^{x=2} x\sqrt{x^2+1}\, dx = \frac{1}{2}\int_2^5 \sqrt{u}\, du = \frac{1}{2}\int_2^5 u^{1/2}\, du$$

$x(x^2+1)^{1/2}$

$$= \frac{1}{2}\,\frac{2}{3}\, u^{3/2} = \frac{1}{3} u^{3/2} \Big|_{2=u}^{5=u}$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$x\,dx = \frac{1}{2}\,du$$

$$\frac{d}{dx}x^n = n x^{n-1} \qquad = \frac{1}{3}\left(5^{3/2} - 2^{3/2}\right)$$

$$\int n x^{n-1}\,dx = x^n + C$$

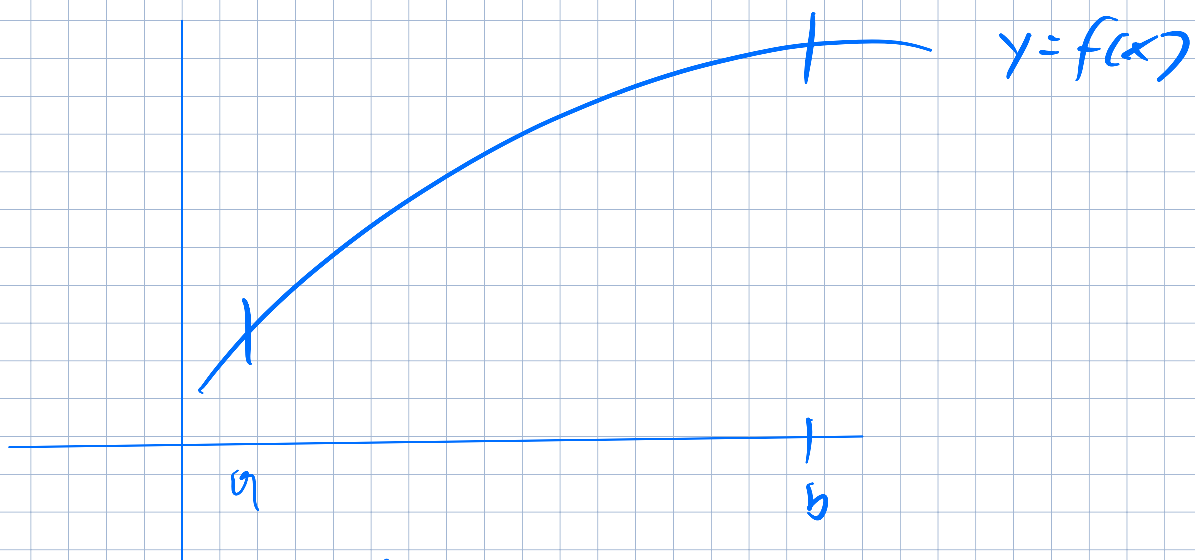
$$\int (n+1) x^n\,dx = x^{n+1} + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$I = \frac{2}{3}\cdot\frac{1}{2}\left(x^2+1\right)^{3/2}\Big|_1^2 = \frac{1}{3}\left(5^{3/2} - 2^{3/2}\right)$$

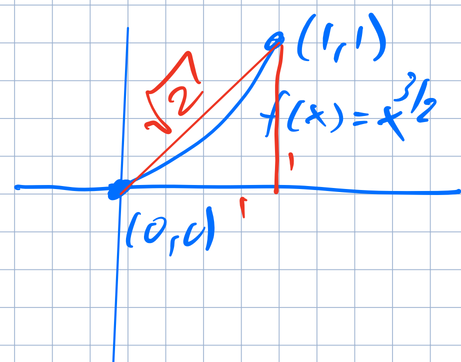
cw

$$1/3 \quad \frac{1}{3} = \frac{2}{3}\cdot\frac{1}{2} = \frac{2\cdot 1}{3\cdot 2} = \frac{2}{6} = \frac{1}{3}$$



$$L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$$

$$[a, b] = [0, 1]$$



$$f(x) = x^{3/2}$$

$$f'(x) = \frac{3}{2}x^{1/2}$$

$$L = \int_0^1 \left(1 + \frac{9}{4} x\right)^{1/2} dx$$

$$= \frac{4}{9} \cdot \frac{2}{3} \left(1 + \frac{9}{4} x\right)^{3/2} \Big|_0^1 = \frac{8}{27} \left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$$

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air → ballon $3\ f^3/s$

how fast is the radius growing $r' = ?$
when $r = 2'$

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi \cdot 8$$

$$0 = V' = 0$$

$$V' = \frac{4}{3} \pi\, 3 r^2 r' = 4\pi r^2 r'$$

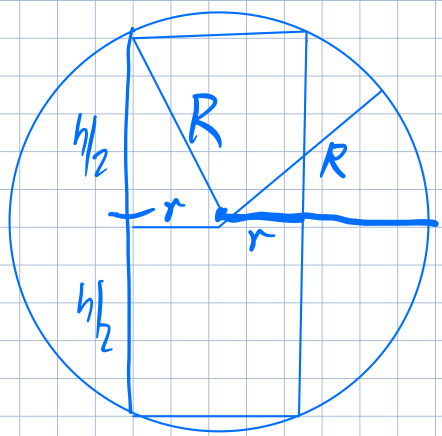
$$r' = \frac{V'}{4\pi r^2}$$

$T = 1$

$V' = 3$

$$r' = \frac{3}{4\pi 4} = \frac{3}{16\pi}$$

Largest Cylinder in a sphere



$$V = \pi r^2 h = \max$$

$$\frac{h^2}{4} + r^2 = R^2 \qquad h = \sqrt{4(R^2 - r^2)}$$

$$r^2 = R^2 - \frac{h^2}{4}$$

$$r = \sqrt{R^2 - \frac{h^2}{4}}$$

$$V = \pi\left(R^2 - \frac{h^2}{4}\right) h$$

$$= \pi\left(R^2 h - \frac{h^3}{4}\right) \qquad ' = \frac{d}{dh}$$

$$V' = \pi\left(R^2 - \frac{3h^2}{4}\right) = 0$$

$$\frac{3h^2}{4} = R^2$$

$$h^2 = \frac{4}{3} R^2$$

$$h = \frac{\sqrt{4}}{\sqrt{3}} R = \frac{2}{\sqrt{3}} R$$

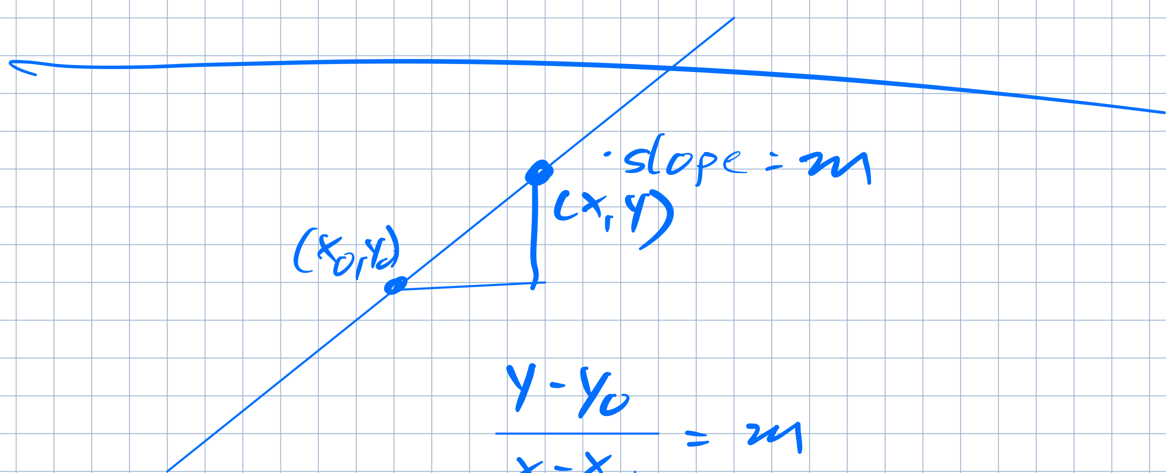
$$\frac{h}{2} = \frac{1}{\sqrt{3}} R \qquad h = \frac{2}{\sqrt{3}} R$$

$$r = \sqrt{R^2 - \left(\frac{h}{2}\right)^2} = \sqrt{R^2 - \frac{R^2}{3 \cdot 4}}$$

$$= \sqrt{R^2 - \frac{1}{3} R^2} =$$

$$= \sqrt{\frac{2}{3} R^2} = \frac{\sqrt{2}}{\sqrt{3}} R$$

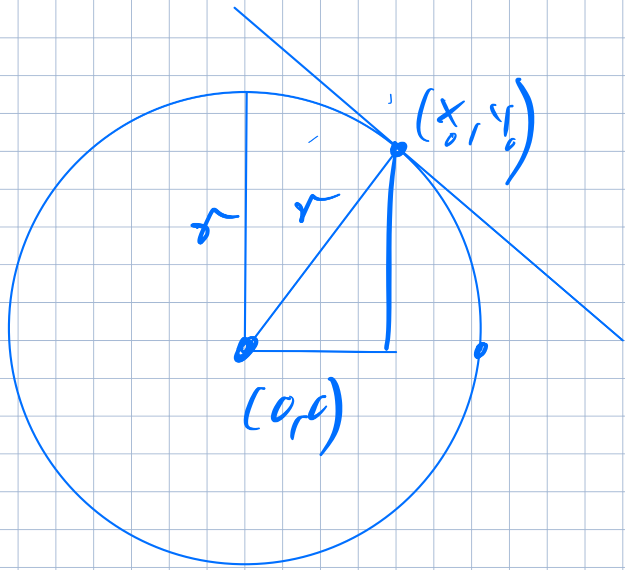
Tangent Line



$$\frac{y - y_0}{x - x_0} = m$$

$$y - y_0 = m(x - x_0)$$

$$y = y_0 + m(x - x_0)$$



$$y = \sqrt{r^2 - x^2}$$

$$x^2 + y^2 = r^2$$

$$y = y(x)$$

$$x^2 + y^2 = r^2 \qquad y = y(x) \qquad \Big| \frac{d}{dx}$$

$$2x + 2y\frac{dy}{dx} = 2x + 2yy' = 0$$

$$y' = -\frac{x}{y}$$

Tangent $\dfrac{y - y_0}{x - x_0} = -\dfrac{x_0}{y_0}$

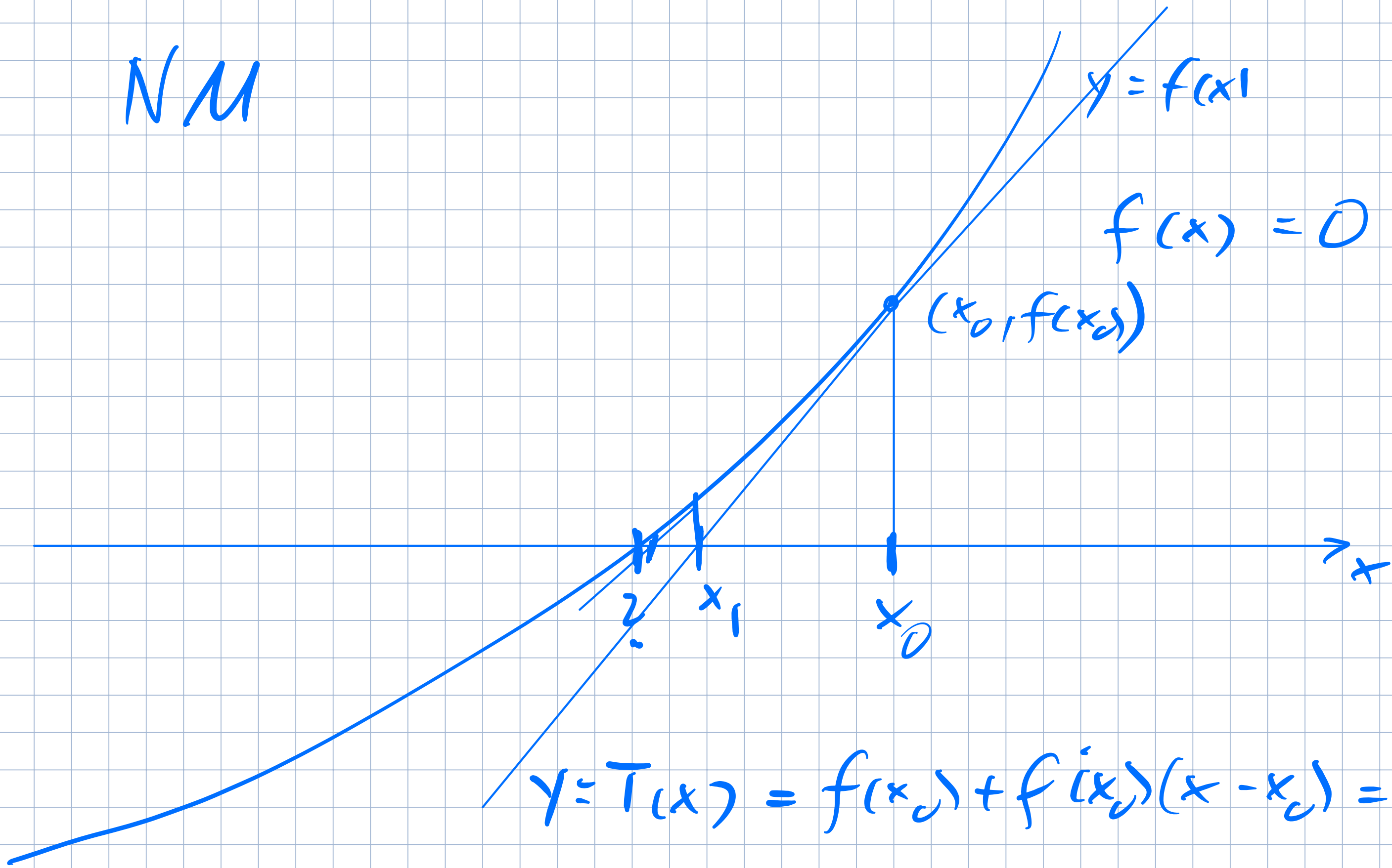
$$y = y_0 - \frac{x_0}{y_0}(x - x_0)$$

$$y = (r^2 - x^2)^{1/2}$$

$$y' = \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x)$$

$$= -\frac{x}{(r^2 - x^2)^{1/2}} = -\frac{x}{y}$$

NM



$f(x) = 0$

$$y = T(x) = f(x_0) + f'(x_0)(x - x_0) = 0$$

$$f'(x_0)(x - x_0) = -f(x_0)$$

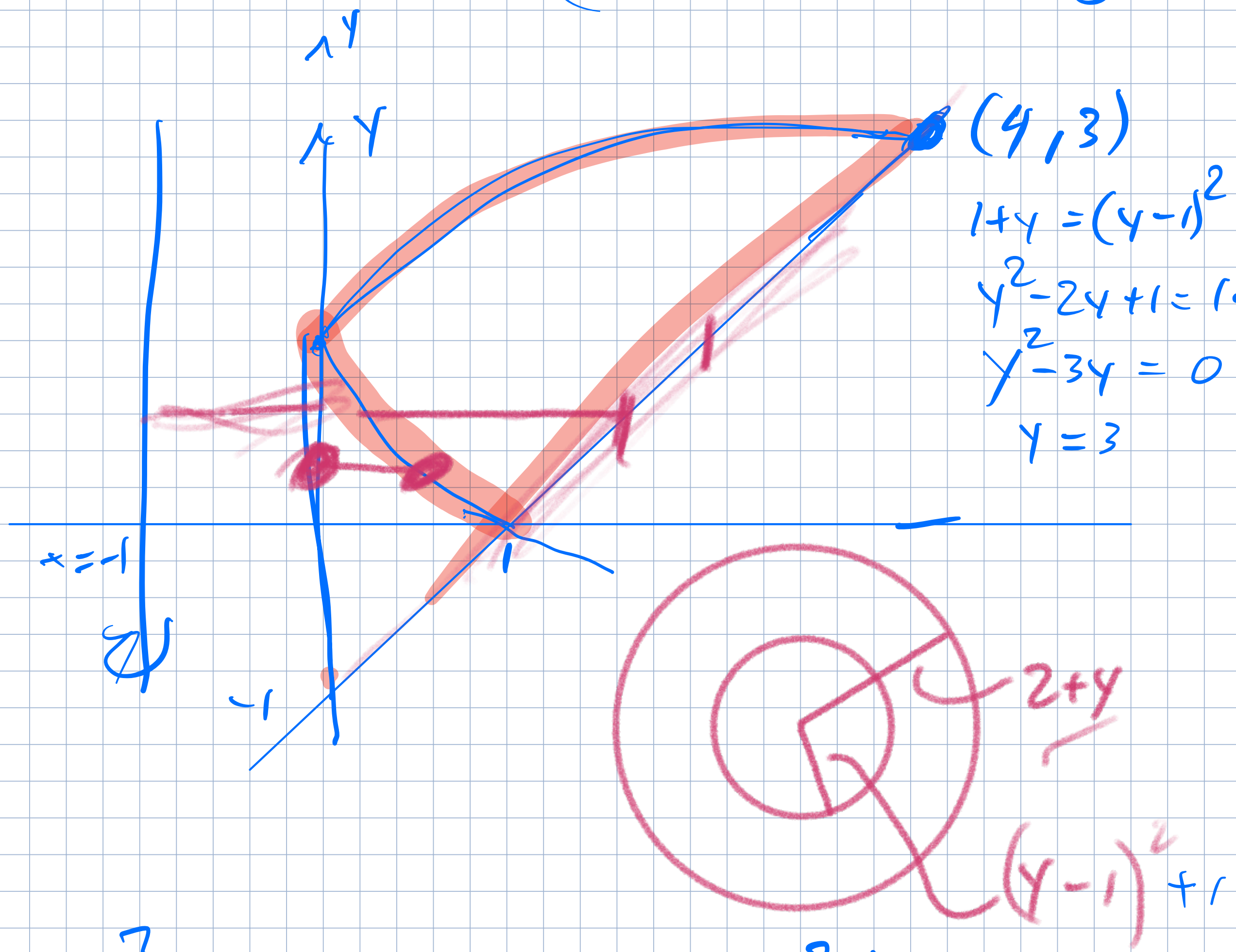
$$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)} = x_1$$

$$x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$x = 1+y \qquad y = x-1 \qquad x = (y-1)^2 \qquad x = -1$



$1+y = (y-1)^2$

$y^2 - 2y + 1 = 1+y$

$y^2 - 3y = 0$

$y = 3$

$$V = \int_0^3 \pi\left((2+y)^2 - \left((y-1)^2+1\right)^2\right) dy$$