

# Review

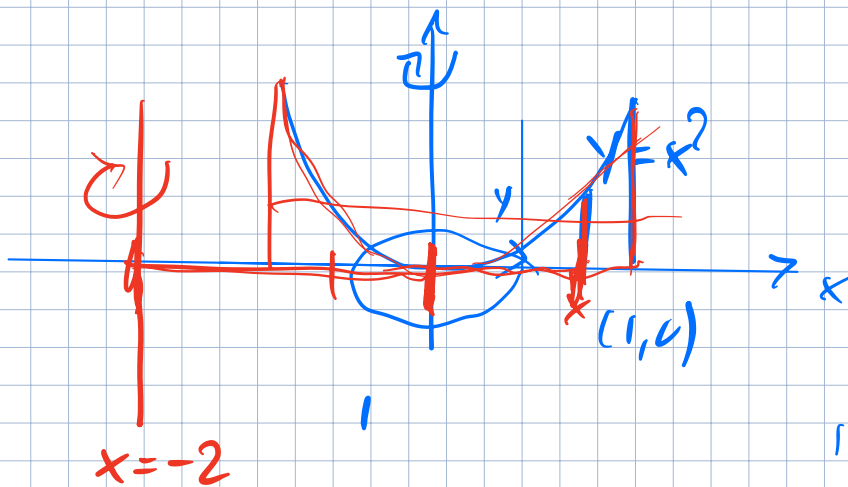
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$$f(x) = x^2$$

x-axis  
 $-1 \leq x \leq 1$



$$\int_0^1 y \cdot 2\pi x \, dx = \int_0^1 2\pi x^3 \, dx$$

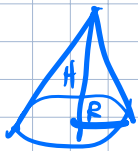
$$= 2\pi \left[ \frac{x^4}{4} \right]_0^1$$

$$= \frac{\pi}{2}$$

$$V = \int_{-1}^1 y \cdot 2\pi(2+x) \, dx = 2\pi \int_{-1}^1 x^2(2+x) \, dx$$

$$\begin{aligned}
&= 2\pi \int_{-1}^1 (2x^2 + x^3) dx \\
&= 2\pi \left[ \frac{2x^3}{3} + \frac{x^4}{4} \right]_{-1}^1 \\
&= 2\pi \left[ \frac{2}{3} + \frac{1}{4} - \left( -\frac{2}{3} + \frac{1}{4} \right) \right] \\
&= 2\pi \frac{4}{3} = \frac{8\pi}{3}
\end{aligned}$$

Differentiation



$$V = \frac{\pi R^2 H}{3}$$

error in R

$$\frac{dV}{dR} = \frac{2\pi R H}{3}$$

$$dV \approx \frac{2}{3} \pi R H \cdot dR$$

$$\frac{dV}{V} = \frac{dV}{\frac{\pi R^2 H}{3}} = \frac{\frac{2}{3} \pi R H}{\frac{\pi R^2 H}{3}} dR$$

$y = f(x)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$\Delta y \approx f'(x) \Delta x$$

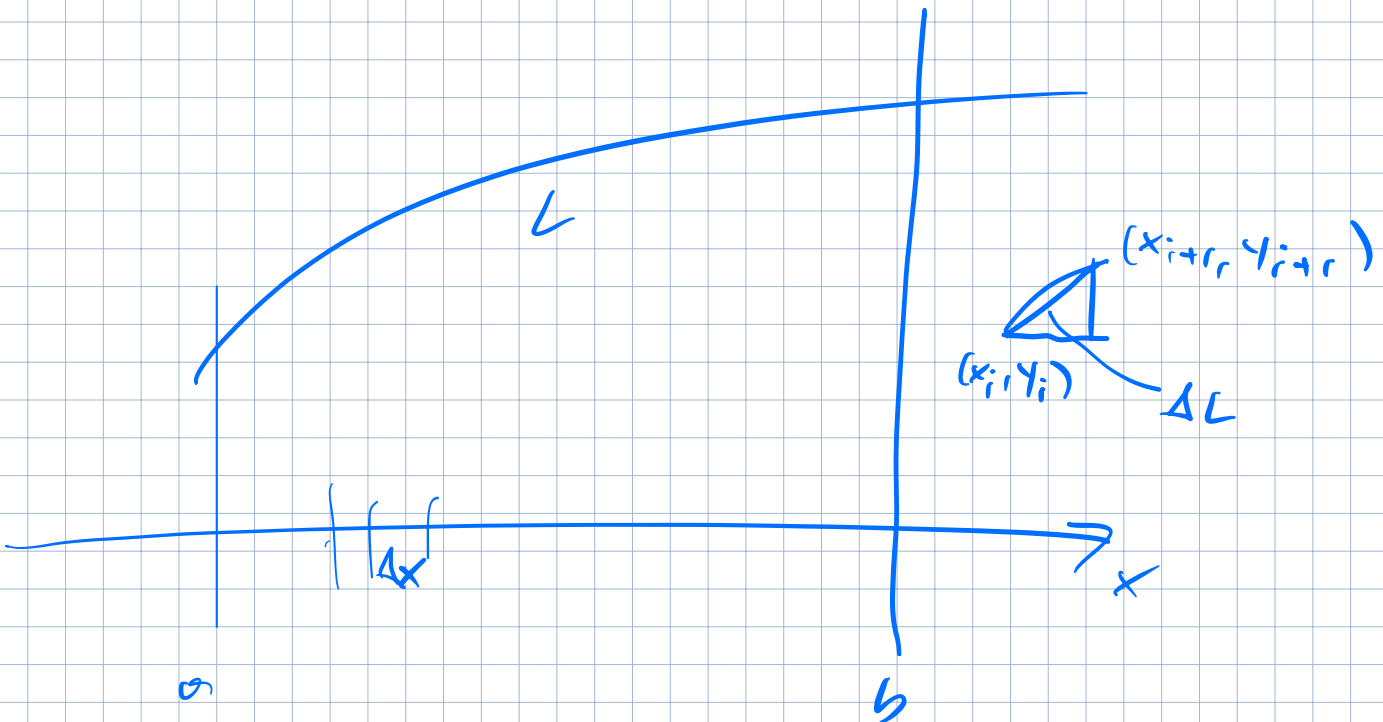
$$dy \approx f'(x) dx$$

$$\frac{dV}{V} = 2 \frac{dR}{R}$$

Height?  $\frac{dV}{dH} = \frac{\pi R^2}{3}$

$$dV = \frac{\pi R^2}{3} dH$$

$$\frac{dV}{V} = \frac{\frac{dV}{\frac{\pi R^2 H}{3}}}{\frac{\pi R^2 H}{3}} = \frac{\frac{\pi R^2}{3} dH}{\frac{\pi R^2 H}{3}} \quad dH = \frac{dH}{H}$$



$$\Delta L \approx \sqrt{\underbrace{(x_{i+1} - x_i)^2}_{\Delta x^2} + (y_{i+1} - y_i)^2}$$

$$y_{i+1} - y_i = f(x_{i+1}) - f(x_i)$$

$$\approx f'(x_i)(x_{i+1} - x_i)$$

$$= f'(x_i) \Delta x$$

$$\rightarrow = \sqrt{\Delta x^2 + f'(x_i)^2 \Delta x^2}$$

$$= \sqrt{1 + f'(x_i)^2} \Delta x$$

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

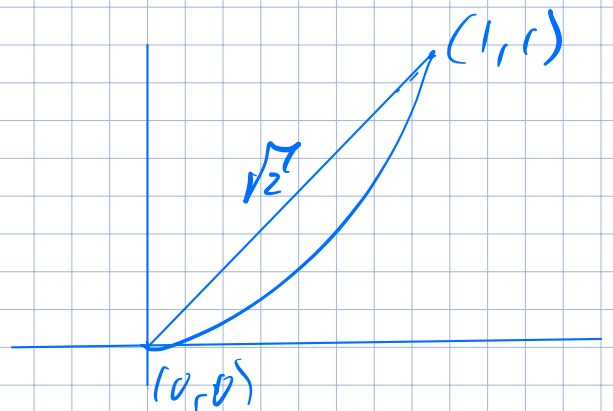
$$[a, b] = [0, 1]$$

$$f(x) = x^{3/2}$$

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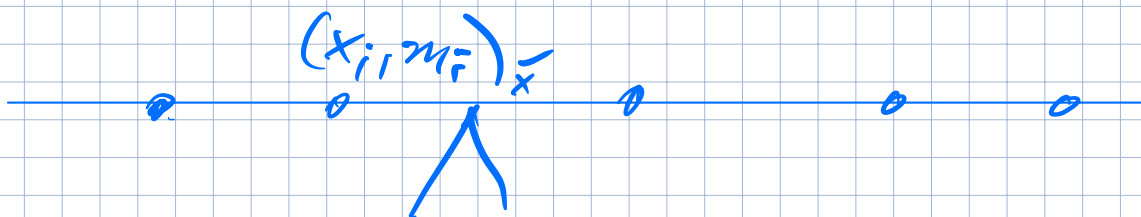
$$f'(x) = \frac{3}{2} x^{1/2}$$

$$f''(x) = \frac{3}{4} x^{-1/2}$$



$$\begin{aligned}
 L &= \int_0^1 \sqrt{1 + \frac{9}{4}x} \, dx \\
 &= \int_0^1 \left(1 + \frac{9}{4}x\right)^{1/2} dx \\
 &= \frac{4}{9} \cdot \frac{2}{3} \left(1 + \frac{9}{4}x\right)^{3/2} \Big|_0^1 \\
 &= \frac{8}{27} \left( \left(\frac{13}{4}\right)^{3/2} - 1 \right)
 \end{aligned}$$

Center of Mass



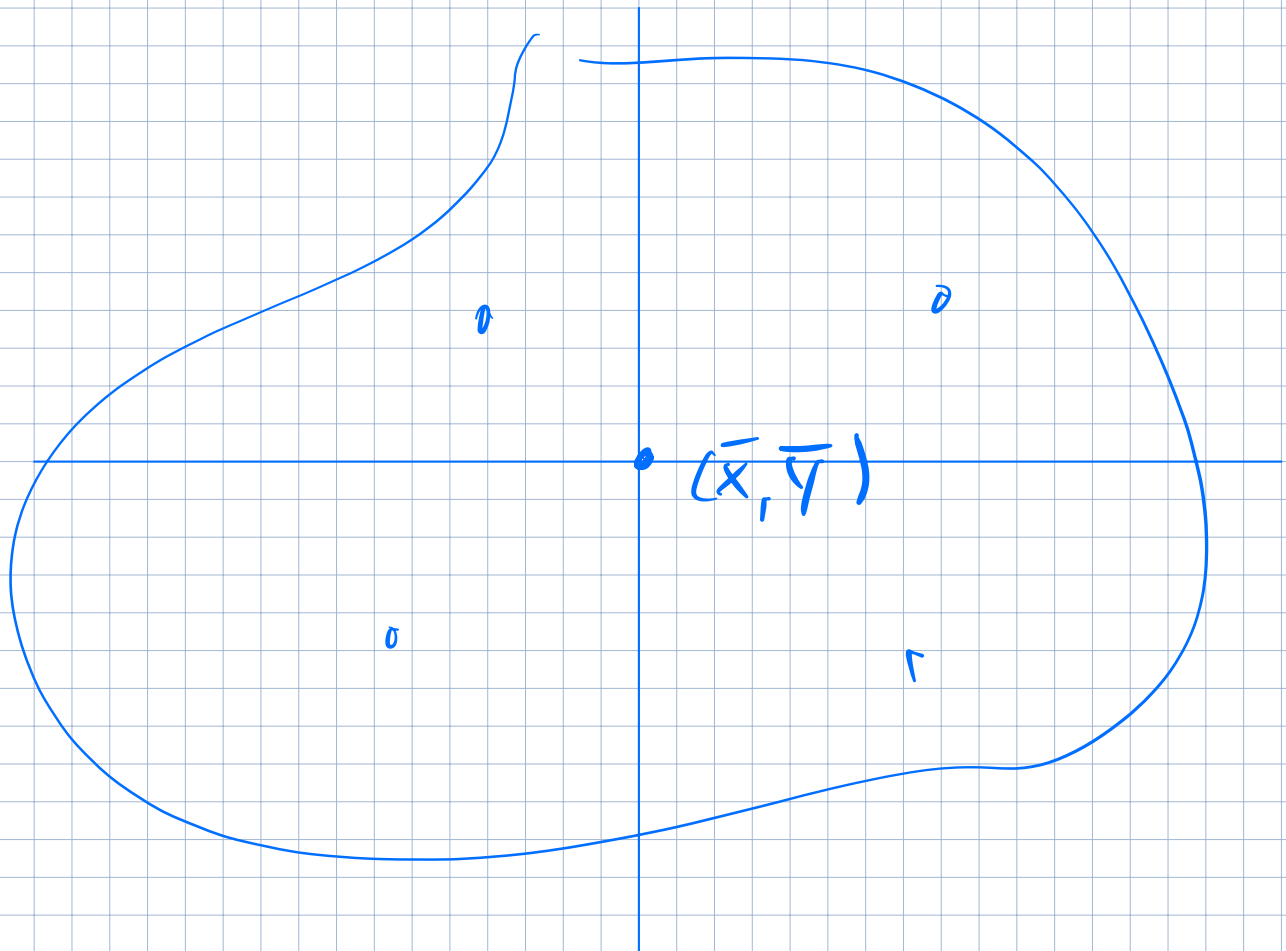
$$\bar{x} = \sum (x_i - \bar{x}) m_i = 0$$

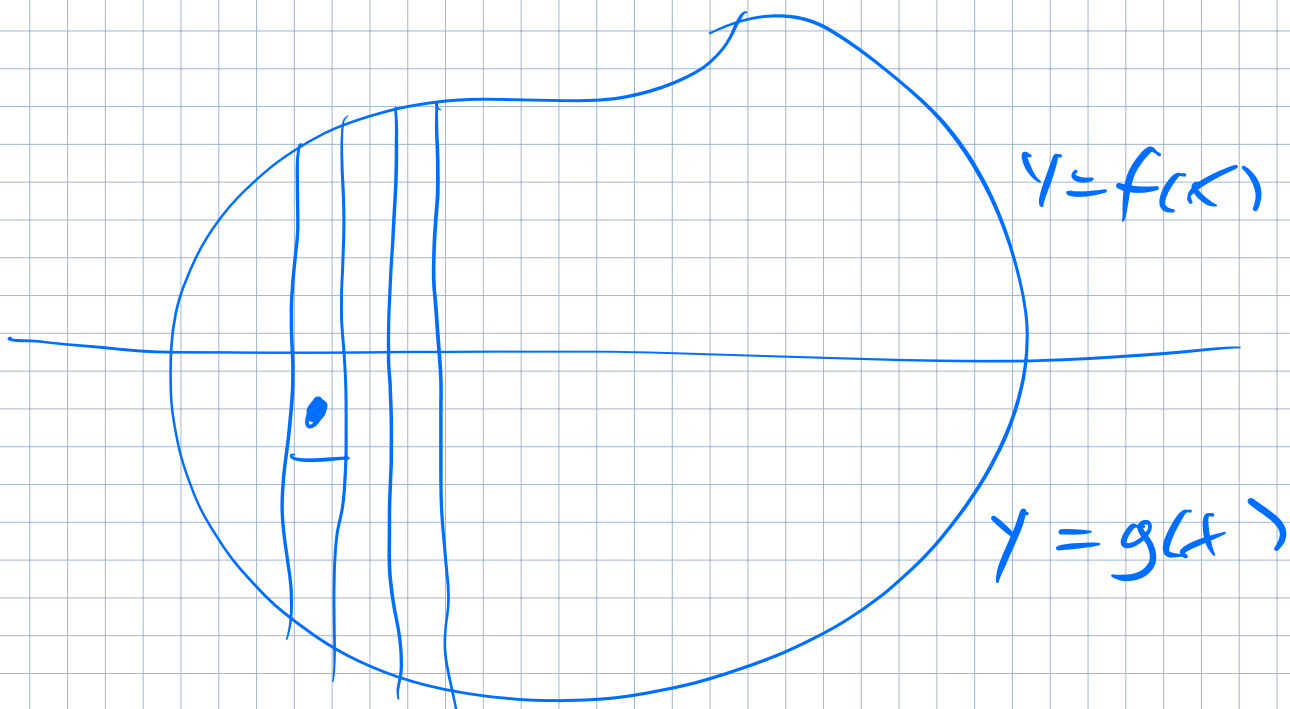
$$\sum x_i m_i = \bar{x} \sum m_i$$

$$\bar{x} = \frac{\sum x_i m_i}{\sum m_i}$$

$\delta(x)$  density

$$\bar{x} = \frac{\int_a^b \delta(x) x dx}{\int_a^b \delta(x) dx}$$





$$\bar{x} = \frac{\int_a^b x(f(x) - g(x)) dx}{\int_a^b (f(x) - g(x)) dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}(f(x) + g(x))(f(x) - g(x)) dx}{\int_a^b (f(x) - g(x)) dx}$$

$$\bar{y} = \frac{1}{2} \frac{\int_a^b (f^2(x) - g^2(x)) dx}{\int_a^b (f(x) - g(x)) dx}$$