5.2-3 More Volumes

- Recall basic idea of volume computation: integrate the area of the cross section in a direction that is perpendicular to the cross section.

- Particularly important are solids of revolution: solid obtained by rotating a two dimensional region around an axis.

- Depending on how we slice and rotate we can get disks, washers, or shells.

- We’ll do more examples today.

- **Example**: Problem 19, page 293. Suppose you drill a (cylindrical) hole of radius $r$ through a sphere of radius $R$ such that the axis of the hole passes through the center of the sphere. Compute the volume of the remaining ring, and the volume of the material that’s been removed.
\[ V_{\text{left}} = \int_0^R 2\pi x \, dx = 4\pi \int_0^R (R^2 - x^2)^{3/2} \cdot x \, dx \]

\[ = 4\pi \cdot \frac{2}{3} \left( \frac{1}{2} \right) (R^2 - x^2)^{3/2} \bigg|_0^R \]

\[ = -\frac{4\pi}{3} \left( R^2 - x^2 \right)^{3/2} \bigg|_0^R \]

\[ = \frac{4\pi}{3} \left( R^2 - r^2 \right)^{3/2} \]

Volume left

\[ V_{\text{gone}} = \frac{4\pi}{3} \left( R^3 - (R^2 - r^2)^{3/2} \right) \]

\[ = \int_0^r -dx \quad = -\frac{4\pi}{3} \left( R^2 - x^2 \right)^{3/2} \bigg|_0^r \]

\[ = -\frac{4\pi}{3} \left( R^2 - r^2 \right)^{3/2} + \frac{4\pi}{3} R^3 \]
**Example:** Compute the volume of the solid obtained by rotating the ellipse

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

around (a) the x-axis, or, (b) the y-axis.

\[
\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \\
\frac{y^2}{} = b^2 \left( 1 - \frac{x^2}{a^2} \right)
\]

\[
V_x = \int_{-a}^{a} \pi y^2 \, dx = b^2 \pi \int_{-a}^{a} \left(1 - \frac{x^2}{a^2} \right) \, dx
\]

\[
= \pi b^2 \left[ x - \frac{x^3}{3a^2} \right]_{-a}^{a}
\]

\[
= \pi b^2 \left( a - \frac{a^3}{3a^2} - \left(-a + \frac{a^3}{3a^2} \right) \right)
\]

\[
= \frac{4\pi a b^2}{3}
\]

\[
V_y = \frac{4\pi a b^2}{3}
\]
Example: Problem 29, page 287. Find the volume of the “+” sign formed by two intersecting cylinders, both of length $L$ and radius $r$, as shown in Figure 1.

$$V = 2\pi r^2 L - V_{\text{intersection}}$$

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from https://i.stack.imgur.com/IUAHL.png, see also Figure 16 on page 287 of the textbook
\[ W = \int_{-r}^{r} dz \]
• Compute the volume of a general cone.