Review 1220-23 4/3/24

$$F' = f$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\Big(F(x) - F(a)\Big)$$

$$= \frac{d}{dx}F(x) = f(x)$$

$$\frac{d}{dx}\int_a^x e^{-t^2}\,dt = e^{-x^2}\cdot\frac{d}{dx}x$$

$$F'(z) = e^{-z^2}$$

$$\frac{d}{dx}\int_a^{\sin x} e^{-t^2}\,dt = \frac{d}{dx}\Big(F(\sin x) - F(a)\Big)$$

$$= F'(\sin x)\cos x =$$

$$= e^{-\sin^2 x}\cos x$$

$$\frac{d}{dx}\int_a^{x^2+\sin x} t^t - e^t \, dt = \left((x^2+\sin x)^{x^2+\sin x} - e^{x^2+\sin x}\right)(2x+\cos x)$$

$$\frac{d}{dx}\int_{L(x)}^{U(x)} f(t)\,dt = \frac{d}{dx}\Big(F(U(x)) - F(L(x))\Big)$$

$$= F'(U(x))U'(x) - F'(L(x))L'(x)$$

$$= f(U(x))\,U'(x) - f(L(x))\,L'(x)$$

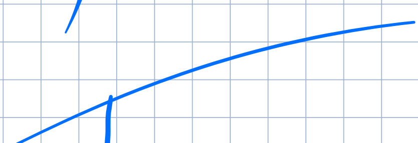
$$\frac{d}{dx}\int_0^{\sin x} t^2\,dt = \frac{d}{dx}\left[\frac{t^3}{3}\right]_0^{\sin x} = \frac{d}{dx}\frac{\sin^3 x}{3}$$

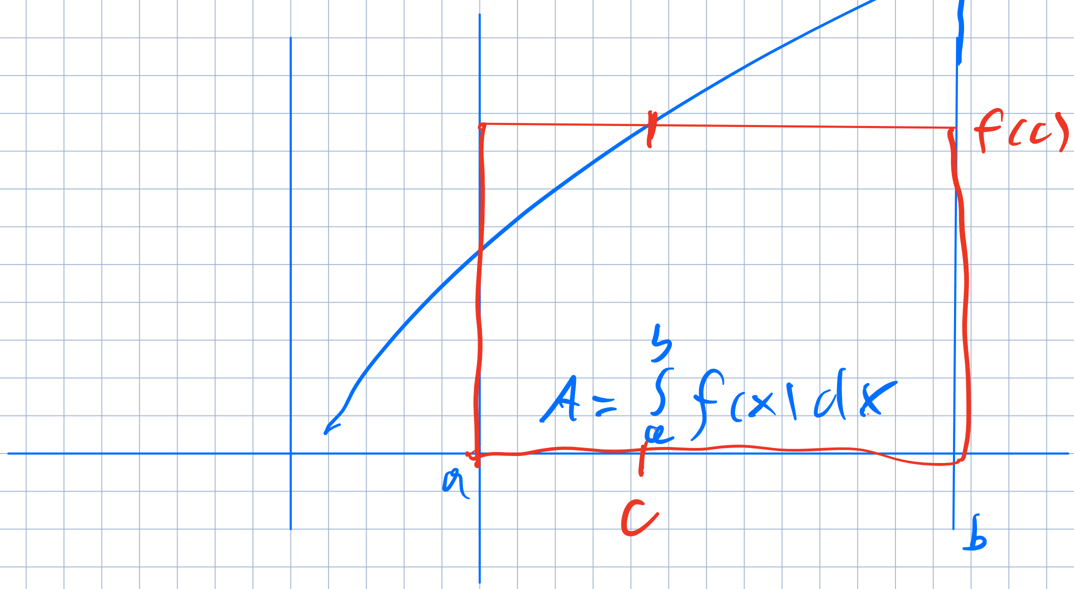
$$= \frac{3\sin^2 x}{3}\cdot\cos x$$

$$= \sin^2 x \cos x$$

MVT integrals

exists c in $[a,b]$ $\quad \int_a^b f(x)\,dx = f(c)(b-a)$





$$x^3+6x^2+3x+1 \qquad [a,b]=[0,1]$$

$$A=\int_0^1 \underbrace{x^3+6x^2+3x+1}_{f(x)}\,dx$$

$$=\left[\frac{x^4}{4}+\frac{6}{3}x^3+\frac{3}{2}x^2+x\right]_0^1=\frac{1}{4}+\frac{6}{3}+\frac{3}{2}+1=S$$

$$\frac{3+24+18+12}{12}$$

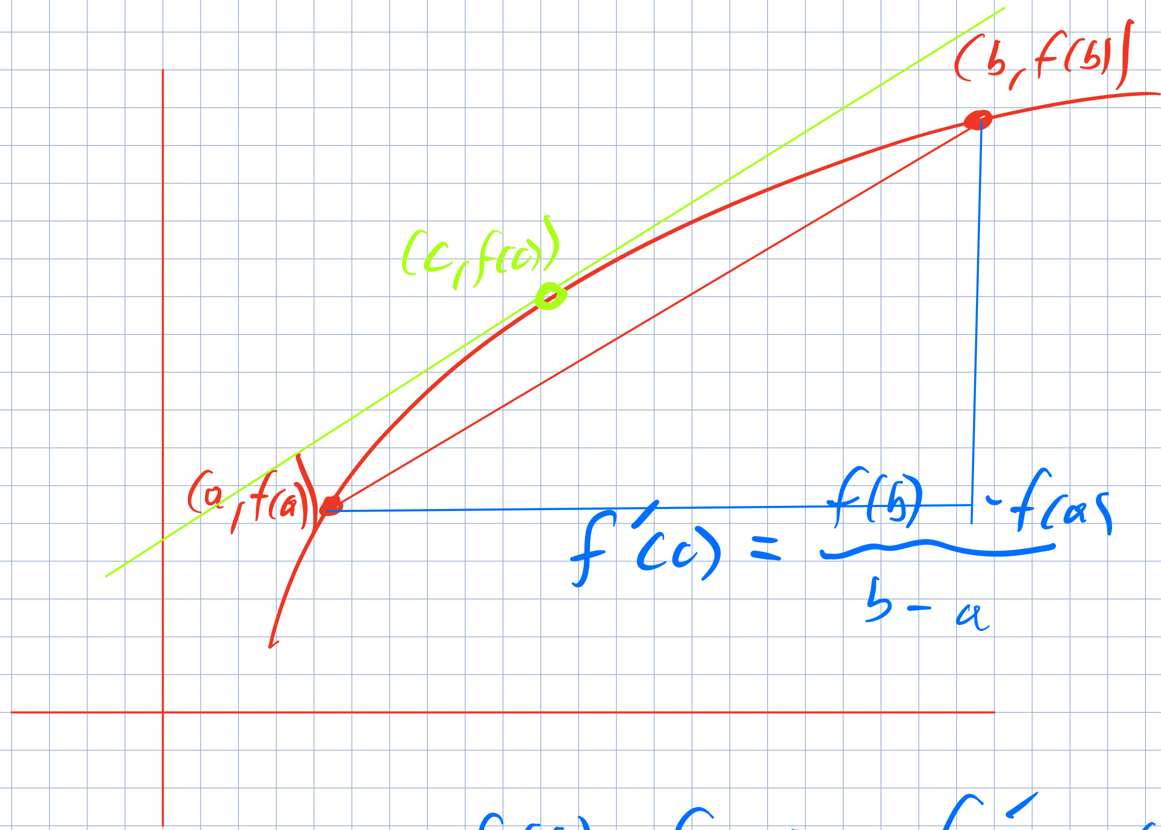
$$=\frac{57}{12}=\frac{19}{3}$$

$$(1-0)f(c)=\frac{19}{3}$$

$$\frac{19}{3}=(1-0)f(c)$$

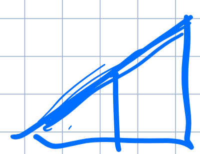
$$c$$

$$(b-a)f(c)=A$$



$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$f(b) - f(a) = f'(c)(b-a)$$



$$f(x) = x^p \qquad [a,b] = [0,1]$$

$$f'(x) = p x^{p-1}$$

$$f(b) - f(a) = 1^p - 0^p = 1 = p\, c^{p-1}(1-0) =$$

$$p\, c^{p-1} = 1$$

$$c^{p-1} = \frac{1}{p}$$

$$c = \left(\frac{1}{p}\right)^{1/(p-1)}$$

$$1+2+3+\dots+n = \sum_{i=1}^{100} i$$

$$a_1+a_2+a_3 \dots + a_n = \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{3} i^2 = 1+4+9 = 14$$

$$\sum_{i=0}^{2} \sin i = \sin 0 + \sin 1 + \sin 2$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$k \quad k$

$$\sum k a_i = k \sum a_i$$

Optimization