Math 1210-23 Notes of 1/10/24

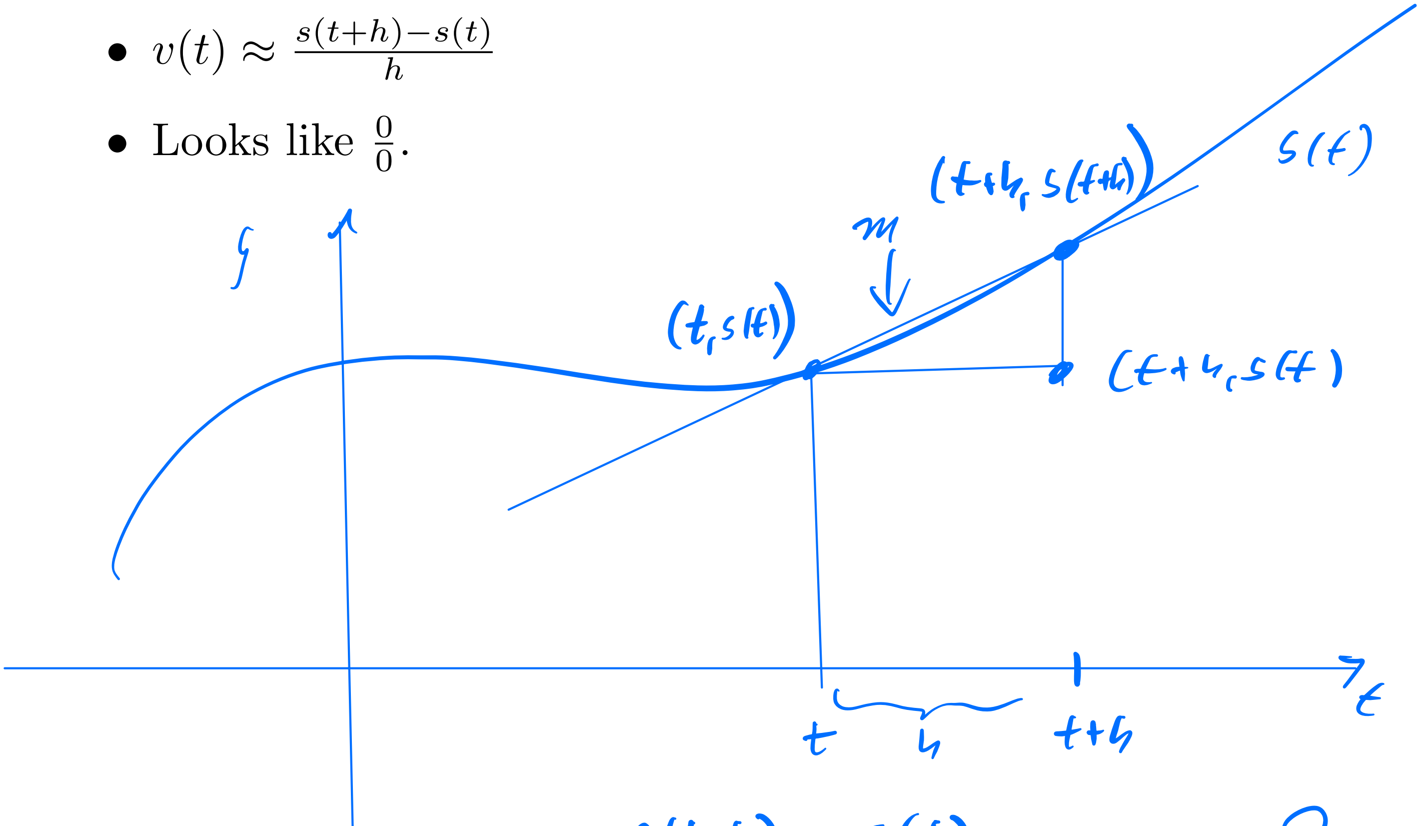
Announcements

- On our Canvas home page you will find the full information about yesterday: The notes, the annotated notes, and video of the lecture.
- Home works usually open on Monday and close 10 days later on Wednesdays. However, hw 1 is open until January 22 (two days before hw 2 closes), since students move between classes during the first couple of weeks of the semester, and some may join our class late.
- Wile the home work is open you can keep returning to it and submit answers. Don't worry when ww tells you the answers have already been submitted. Also, for almost all questions you have an unlimited number of attempts.
- If you won a point in our contest you'll be able to see it in the grades list on Canvas. However, those points will be incorporated only at the end of the semester. In the meantime they do not contribute to your overall percentage showing on Canvas.
- Monday, January 15 is Martin Luther King Day, no class.

- beginning with Calculus proper!
- every day we will have something new
- and every day will build on what we did the previous day.

Make sure you stay on top of things!

- Recall the two tasks we set for ourselves:
- Compute the slope of a tangent:
- $s(t)$ location, $v(t)$ velocity
- $v(t) \approx \frac{s(t+h)-s(t)}{h}$
- Looks like $\frac{0}{0}$.

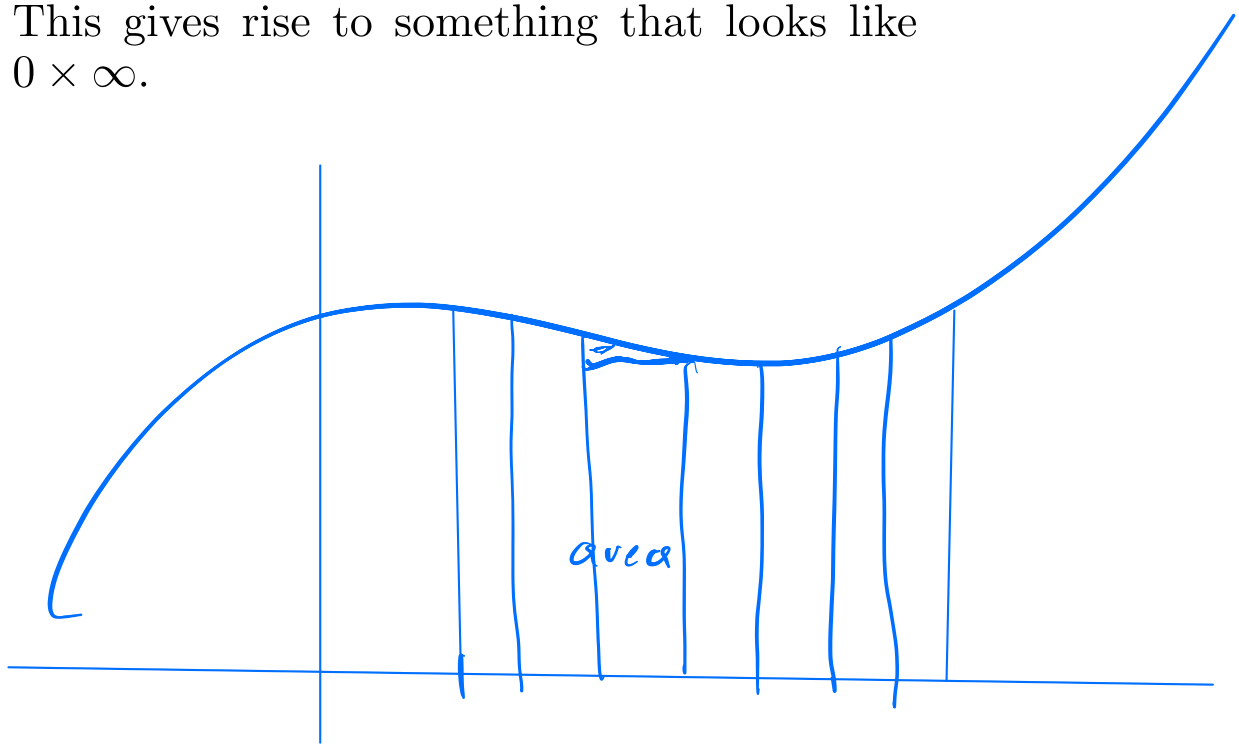


$$m = \frac{s(t+h) - s(t)}{h} \longrightarrow ?$$

$$h \to 0$$

$$\frac{0}{0}$$

- Compute the area underneath the graph of a curve.
- Our approach will be to chop the interval into subintervals, approximate the area in each subinterval by the area of a box, add up the areas of the boxes, and ask what happens as the number of boxes goes to infinity and the size of each box goes to zero.
- This gives rise to something that looks like $0 \times \infty$.



$$0 \cdot \infty$$

1.1 Limits

- to deal with **indeterminate expressions** like $\frac{0}{0}$ and $0 \times \infty$ we need the concept of **limits**.
- That's the contents of section 1.1.
- Example 2, page 57.
- What happens to
$$y = \frac{x^2 - x - 6}{x - 3}$$
as x gets close to 3?

$$\frac{0}{0}$$

- Some numerical values:

x	$\frac{x^2-x-6}{x-3}$
2.9	4.9
2.99	4.99
3.01	5.01
3.1	5.1

- Looks like y gets close to 5.
- We write this as

$$\lim_{x \longrightarrow 3} \frac{x^2 - x - 6}{x - 3} = 5$$

and say *"The limit of $\frac{x^2-x-6}{x-3}$ as x approaches 3 equals 5."*

- Numerically approximating a limit often works pretty well!

- We can also do this analytically, as shown here:

$$\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3} = \lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)}$$

$$= \lim_{x\to 3} (x+2)$$

$$= \left(\lim_{x\to 3} x\right) + 2$$

$$= 3 + 2$$

$$= 5$$

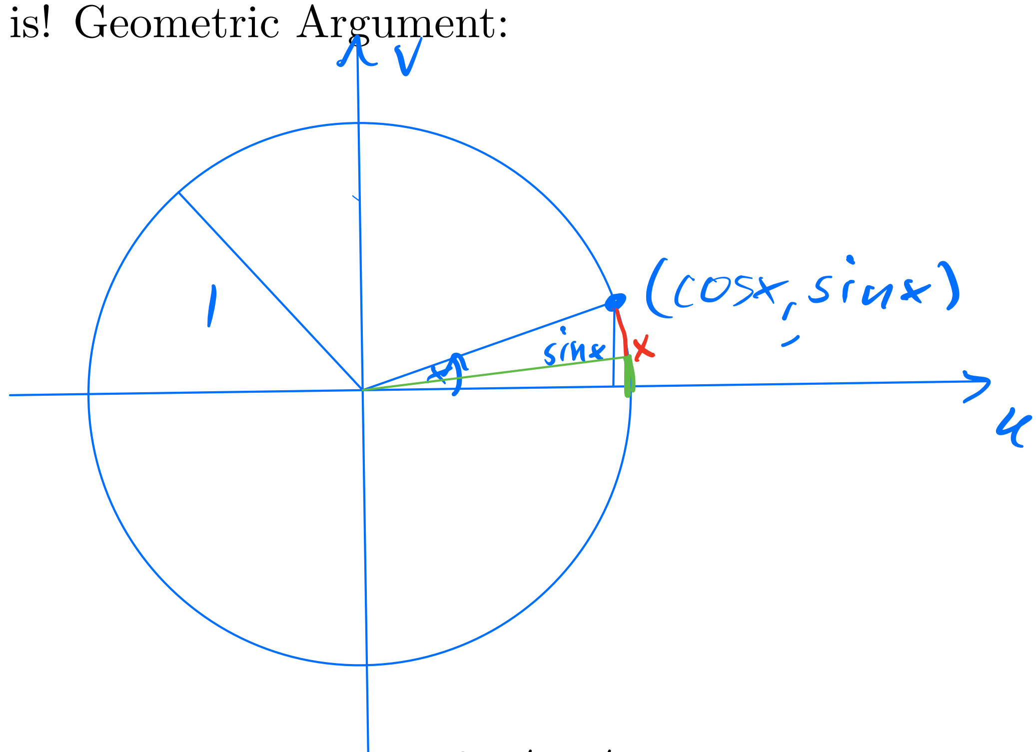
- Example 3:

$$\lim_{x \longrightarrow 0} \frac{\sin x}{x} = ? \quad = 1$$

- Make sure you set your calculator to radian mode!
- Numerical Values:

x	$\frac{\sin x}{x}$
0.1	0.998334167
0.01	0.999983333
0.001	0.999999833

- Query: what about negative x?
- Limit looks like 1.
- It is! Geometric Argument:



Intuitive Meaning of Limit

$$\lim_{x \longrightarrow c} f(x) = L$$

means that $f(x)$ gets arbitrarily close to L as x gets sufficiently close to c.

- We can make $f(x)$ as close to to L as we wish. All we have to do is pick x as close to c as we have to.
- Major point: It does not matter what happens when $x = c$. $f(c)$ may be undefined, or it could be some value other than L.
- The limit may be obvious:

  Example: $\lim_{x \longrightarrow 3}(4x - 5) =$ $4\left(\lim_{x \to 3} x\right) - 5 = 4 \cdot 3 - 5 = 7$
- In this example, nothing goes wrong at $x = 3$. But of course we may be in trouble with expressions like $\frac{0}{0}$ or $0 \times \infty$.
- Numerical evaluation does not always tell the limit, and of course it fails when there are parameters in the problem.
- Example: we computed yesterday

$$\lim_{h \longrightarrow 0} \frac{16(t+h)^2 - 16t^2}{h} = 32t.$$

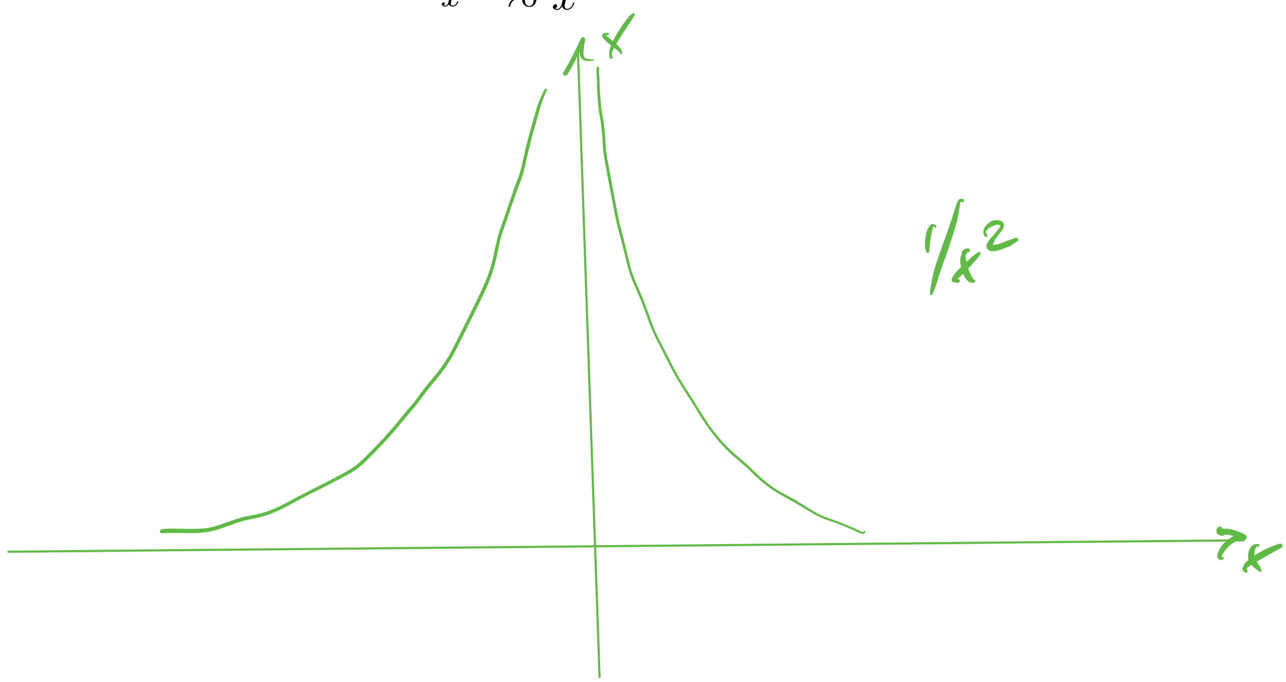
- For example, suppose you measure angles in degrees. (Set your calculator to degrees mode.)
- What is
$$\lim_{x \longrightarrow 0} \frac{\sin x^\circ}{x}?$$
- Numerically, we get

x	$\frac{\sin x^\circ}{x}$
0.1	0.017453284
0.01	0.017453292
0.001	0.017453293

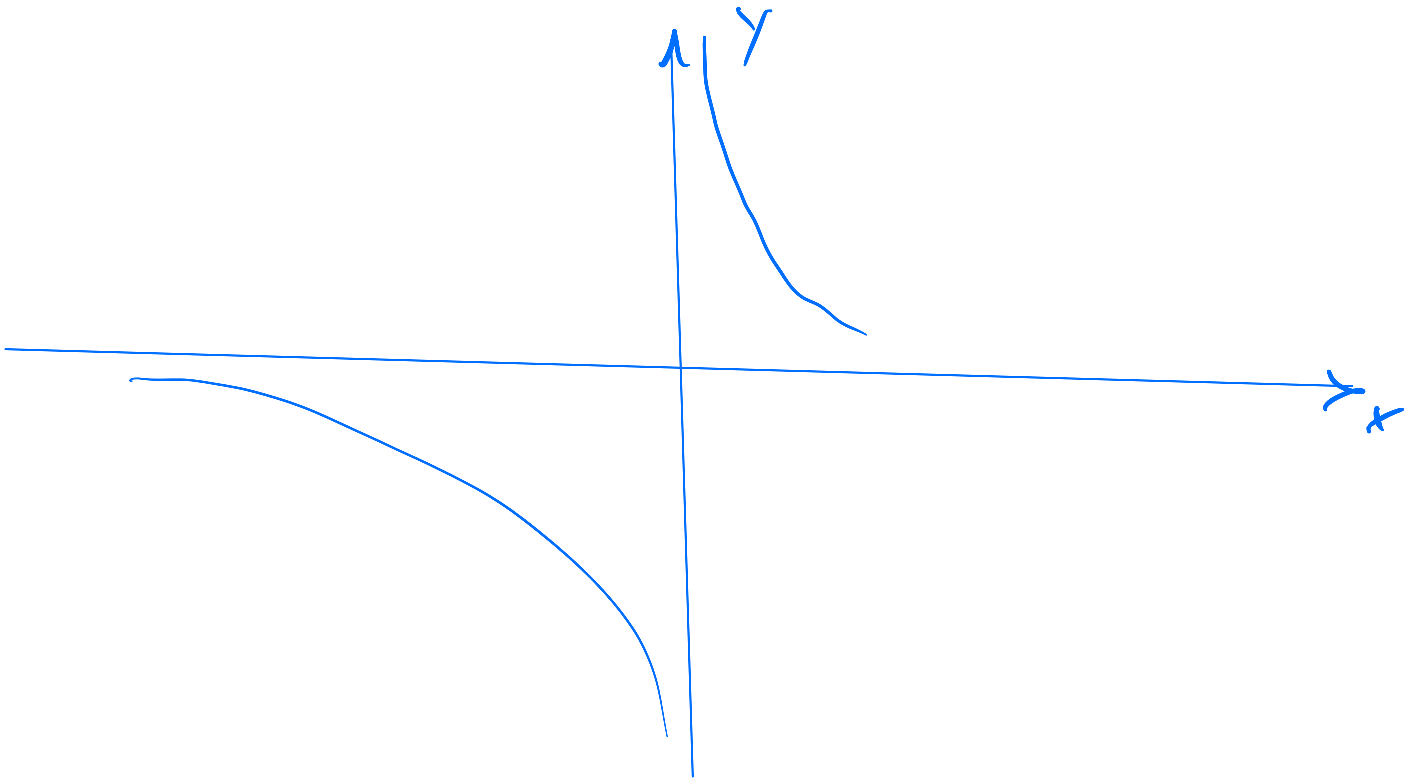
- (Somewhat tricky) exercise: Show that
$$\lim_{x \longrightarrow 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$$

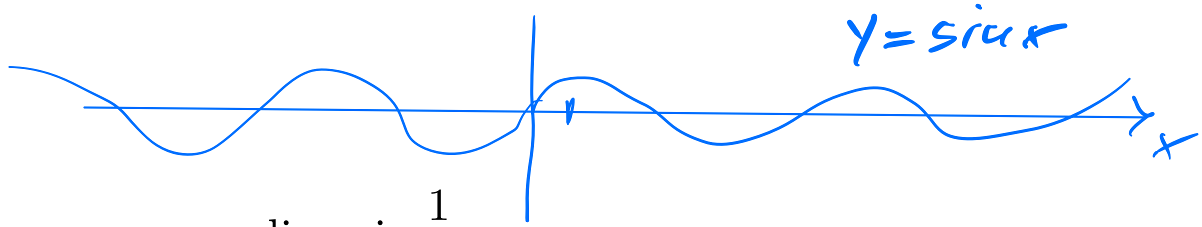
Examples of Non-existent Limits

$$\lim_{x \longrightarrow 0} \frac{1}{x^2}. = \infty$$

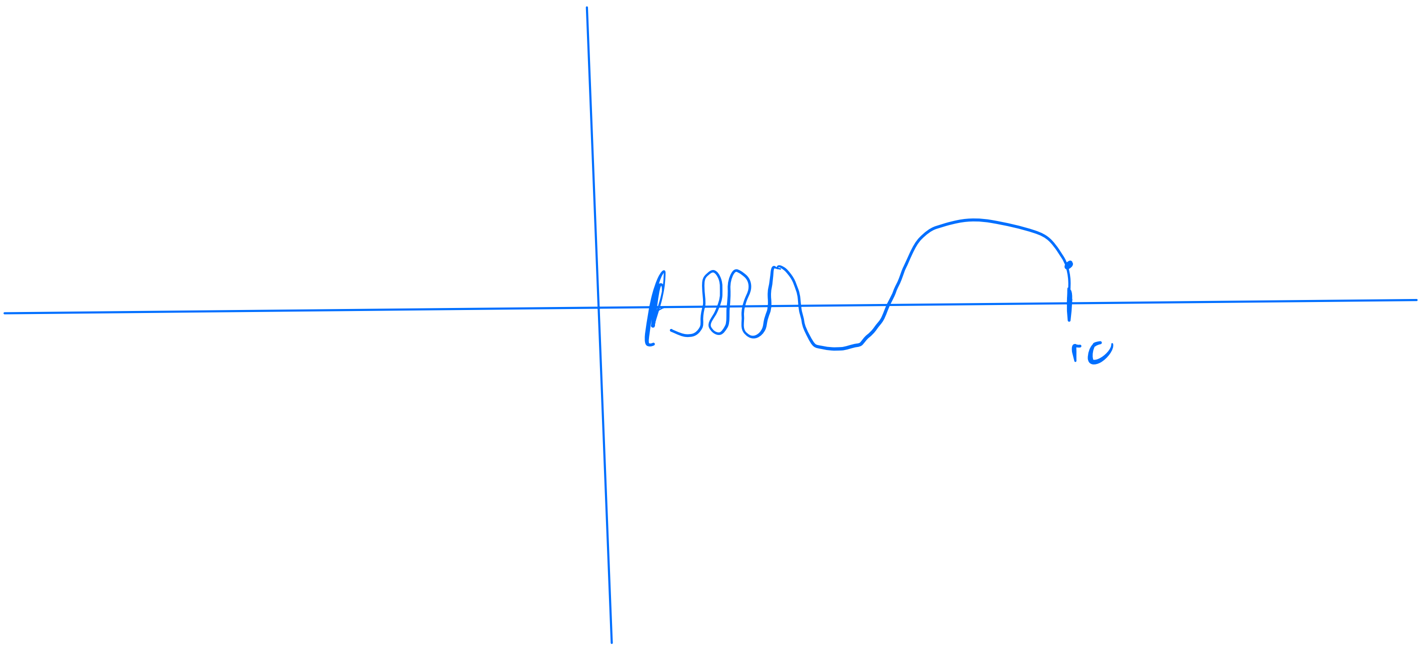


$$\lim_{x\longrightarrow 0} \frac{1}{x}.$$





$$\lim_{x \longrightarrow 0} \sin \frac{1}{x}.$$



- Graph is impossible to draw.

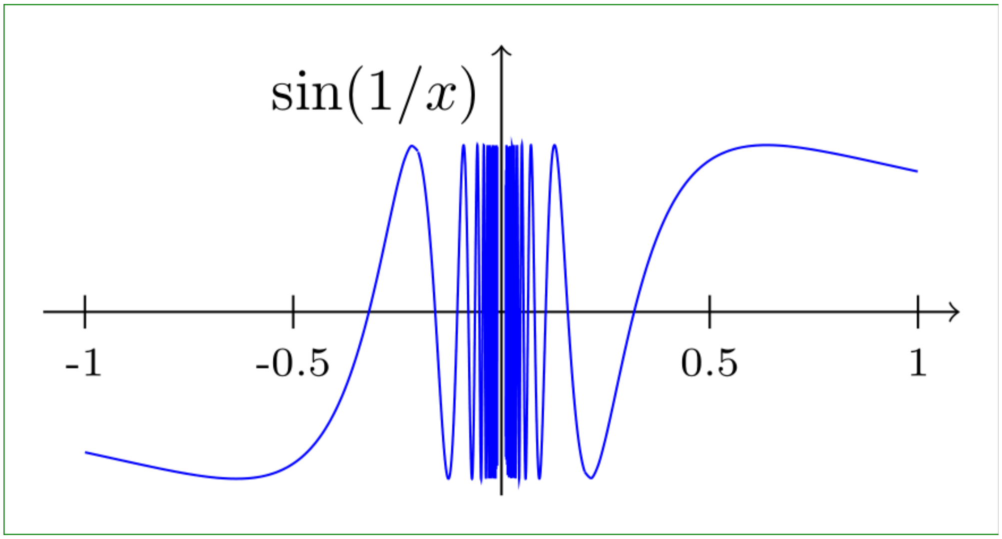


Figure 1. "Graph" of $f(x) = \sin \frac{1}{x}$.

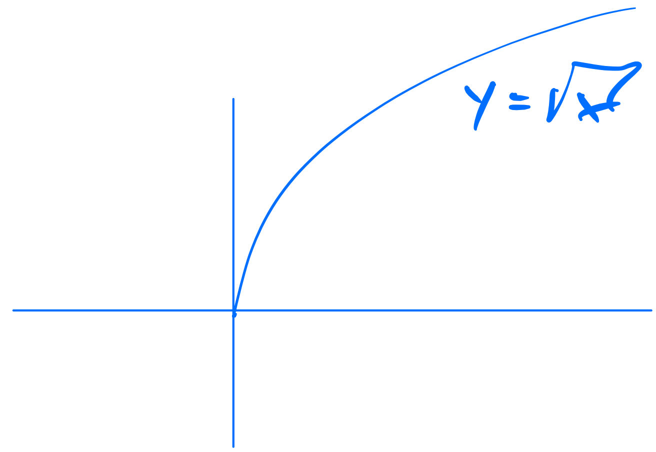
from `https://i.stack.imgur.com/c0gVv.png`

See also Figure 9, page 58, textbook.

One sided Limits

- Example:

$$\lim_{x\longrightarrow 0^+} \sqrt{x} = 0.$$



- Greatest Integer Function

$$[\![x]\!] = \text{ the greatest integer } \leq x.$$

- For example:

$$[\![3.1]\!] = 3$$

$$[\![2.9]\!] = 2$$

$$\frac{1}{x} \cdot (2x) = 2$$

$$[\![3]\!] = 3$$

$$x \to 0$$

$$[\![-3.1]\!] = -4$$

$$\frac{1}{x} \cdot 3x = 3$$

$$\lim_{x\longrightarrow 2^+} [\![x]\!] = 2$$

$$\lim_{x\longrightarrow 2^-} [\![x]\!] = 1$$

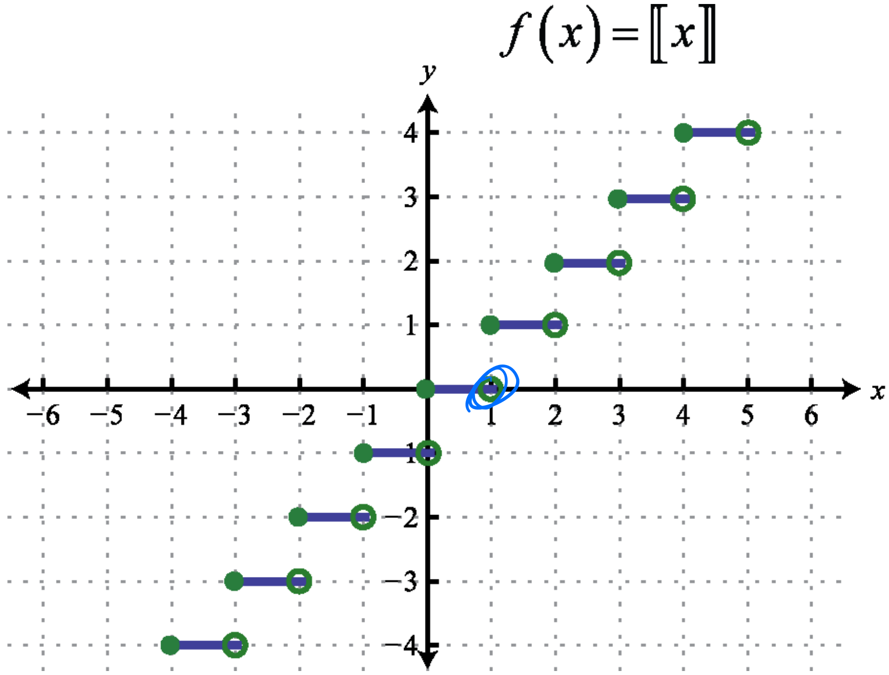


Figure 2. Graph of $f(x) = [\![x]\!]$.

from `http://jennarocca.com/greatest-integer-function-equation/`