Math 1210-23

Notes of 3/12/24

3.5 Sophisticated Graphing

- **Unsophisticated Graphing:** compute a bunch of points on the graph and connect them somehow. That's what your calculator does.
- **Sophisticated Graphing:** Use symmetry, asymptotic behavior, first and second derivatives.
- Quick review of the relationship between derivatives and the shape of the graph of $y = f(x)$:

$f' > 0$	$\longleftrightarrow$	f is increasing
$f' = 0$	$\longleftrightarrow$	horizontal tangent
$f' < 0$	$\longleftrightarrow$	f is decreasing
$f' > 0 \longrightarrow f' < 0$	$\longleftrightarrow$	local maximum
$f' < 0 \longrightarrow f' > 0$	$\longleftrightarrow$	local minimum
$f'' > 0$	$\longleftrightarrow$	f is concave up
$f'' < 0$	$\longleftrightarrow$	f is concave down
f'' changes sign	$\longleftrightarrow$	inflection point

- We have already exploited these ideas and will again in the appropriate context.
- Let's discuss Example 2 from the textbook.

$$
\begin{aligned}
f(x) &= \frac{x^2 - 2x + 4}{x - 2} \\
&= x + \frac{4}{x - 2} \\
f'(x) &= \frac{(2x - 2)(x - 2) - (x^2 - 2x + 4)}{(x - 2)^2} \\
&= \frac{x^2 - 4x}{(x - 2)^2} \\
&= \frac{x(x - 4)}{(x - 2)^2} \\
f''(x) &= \frac{(2x - 4)(x - 2)^2 - 2(x - 2)(x^2 - 4x)}{(x - 2)^4} \\
&= \frac{(2x - 4)(x - 2) - 2(x^2 - 4x)}{(x - 2)^3} \\
&= \frac{8}{(x - 2)^3}
\end{aligned}
$$

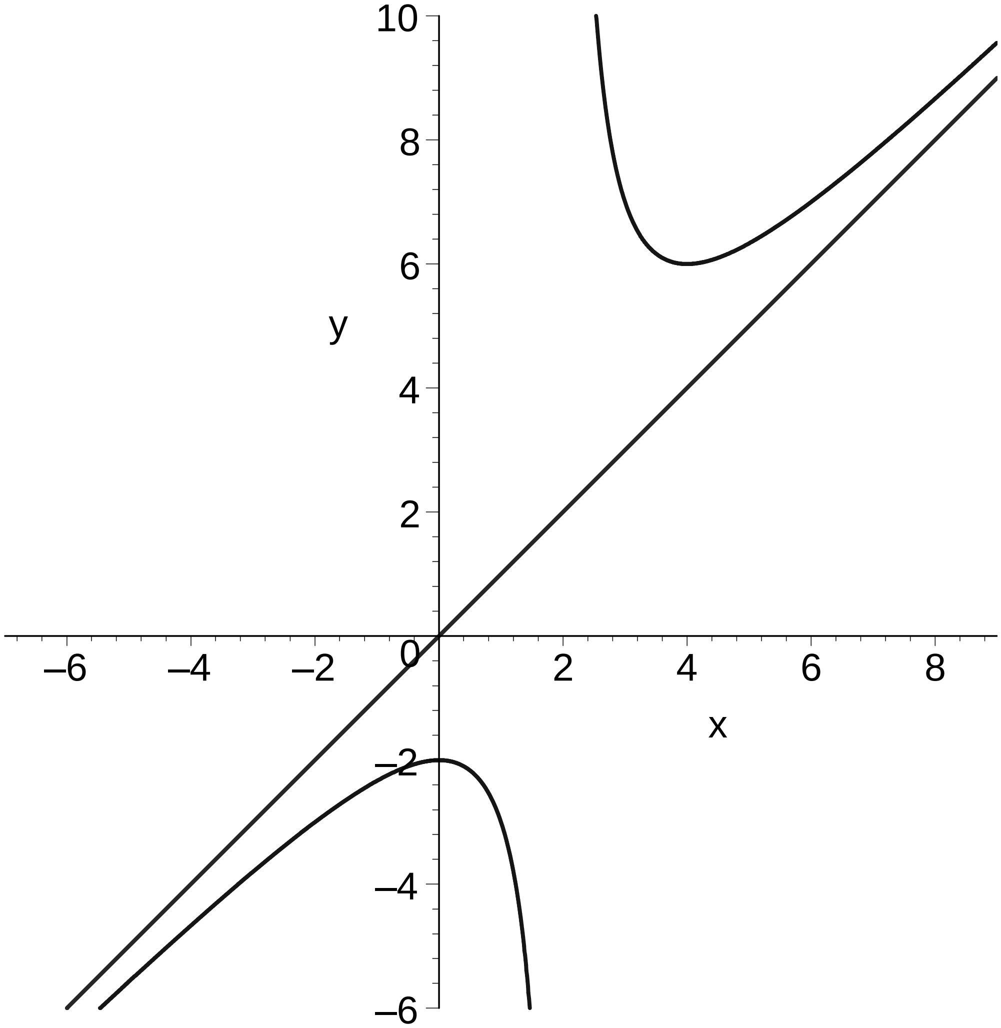

Figure 1. $f(x) = \frac{x^2-2x+4}{x-2}$.

3.6 The Mean Value Theorem (for Derivatives)

There is a point where the tangent is parallel to the secant.

Theorem A, page 186. If f is continuous on $[a, b]$ and differentiable on (a, b) then there is at least one number c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a).$$

- Example: $f(x) = x^3$, $[a, b] = [0, 1]$

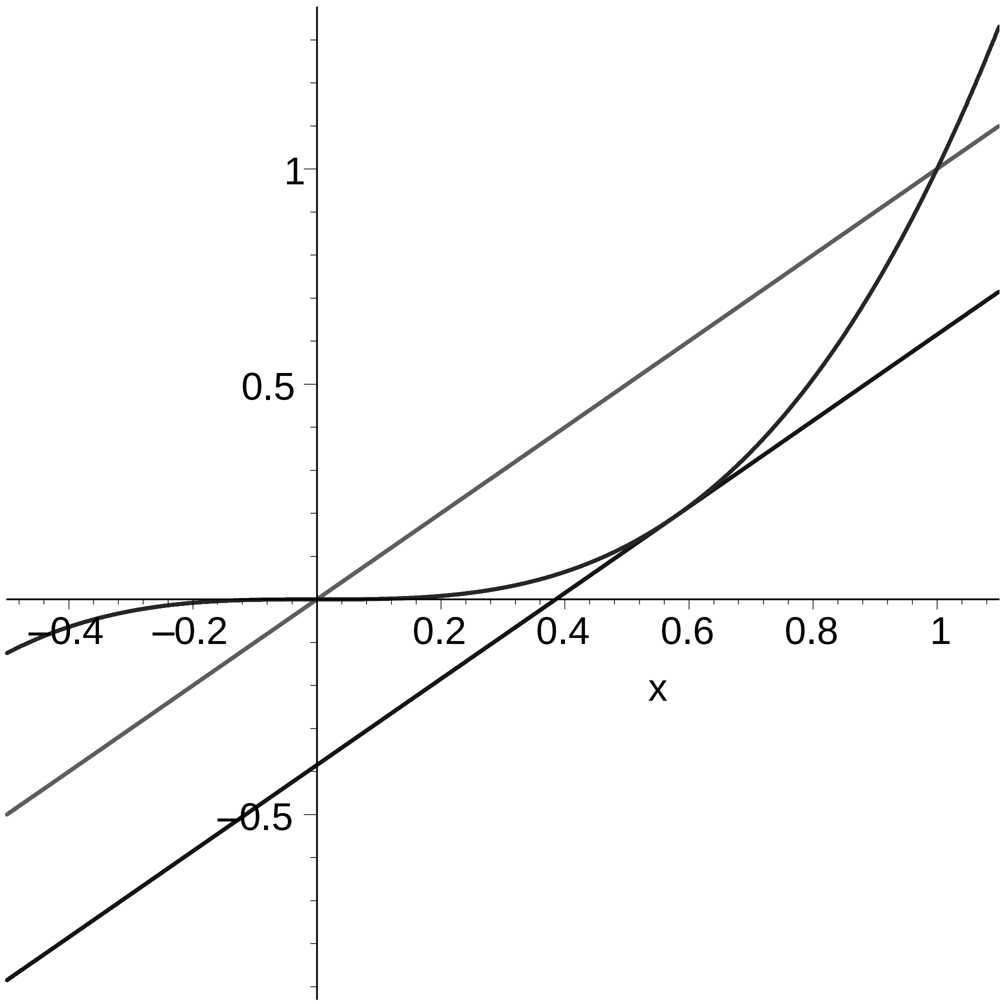

Figure 2. MVT, $f(x) = x^3$, $[a, b] = [0, 1]$, $c = \frac{1}{\sqrt{3}}$.

- Example: $f(x) = x^2$, $[a, b] = [0, 1]$

- Example: f a quadratic polynomial, general interval.

- If f is not differentiable at some point in (a, b) there may be no such c. Example: $f(x) = |x|$ and $[a, b] = [-1, 2]$.

- There may be several several such values of c. Example: $f(x) = \sin x$ and $[a, b] = [0, 1000\pi]$

- It's hard to appreciate the significance of the MVT at this stage but we'll see a nifty application tomorrow.