Math 1210-23 Notes of 1/9/24

Reminders

- Survey due today if possible
- Assessment test due on Thursday
- hw 1 is open, due 1/22
- yesterday’s recording on Canvas
- Today’s annotated notes and recording will be on Canvas some time this afternoon.
What is Calculus?

And why do we need to study it?

- Quick answer to the second question: Because it can be used to solve many problems!

The Key Ideas

- We will introduce the concepts with an example:

Velocity — Location

- Your car has a speedometer (showing current velocity) and an odometer (showing distance covered, or location along a highway).

- Speed versus velocity: velocity has a direction, in this semester just forward and backward, or up and down, distinguished by a plus or minus sign.

- Profound Fact: Velocity and Location are related. One determines the other.

- Think of both as functions of time $t$:
  - $v(t)$ is velocity at time $t$
  - $d(t)$ is location (distance) at time $t$

- If we know our location function $d$ we should be able to compute our velocity function $v$, 
and if we know our velocity function $v$ (and our location at time $t = 0$, say) we should be able to compute our location function.

- Note: we do not just compute location or velocity at a specific time. We know one function for all (relevant) time and compute the other for all (relevant) time.

- This is what Calculus is all about!
  - $v \rightarrow d$: integration
  - $d \rightarrow v$: differentiation

\[
v(t) = 32t \text{ ft/sec}
\]
\[
d(t) = 16t^2
\]

simpler:

\[
v(t) = 100 \text{ mph}
\]
\[
d(t) = 100t
\]

$Vt$ = area under the graph of $v$
\[ d(t) = \frac{1}{2} \ 32t \cdot t = 16t^2 \]
Falling Objects

- simple physical example: falling object, ignore air resistance, consider gravity constant. (It actually does depend on altitude and location on earth.)

- Observation: Velocity increases by 32 ft/sec every second. We say that the acceleration is 32 feet per second squared.

- Let \( v(t) \) be the downward velocity, and assume
  \[ v(0) = 0. \]

- Then clearly
  \[ v(t) = 32t \]

- How far does the object fall in \( t \) seconds?

- In other words, what is \( d(t) \)? (Think of \( d \) as distance or depth.)

- Let’s figure it out.
Going the other way

- Suppose we know the distance. How can we figure out the velocity?

- Let’s apply the ideas to a situation where we already know the answer!

- Suppose

  $$d(t) = 16t^2$$

- What is $v(t)$. Of course we should get

  $$v(t) = 32t$$

  but suppose we don’t know that yet.
\[ m = \frac{16(t+h)^2 - 16t^2}{(t+h) - t} \]

\[ = \frac{16(t+h)^2 - 16t^2}{h} \rightarrow \frac{0}{0} \]

\[ = \frac{16(t^2 + 2th + h^2) - 16t^2}{h} \]

\[ = \frac{16t^2 + 32th + 16h^2}{h} - 16t^2 \]

\[ = \frac{32th + 16h^2}{h} \]

\[ = 32t + 16h \rightarrow 32t \]

\[ \text{as } h \rightarrow 0 \]
simpler

\[ d = 100t \]
The Fundamental Theorem of Calculus

\[
\frac{d(t+h) - d(t)}{h}
\]
The Plan (for both Math 1210 and 1220)

- Make the idea $h \to 0$ precise. (This will give rise to the concept of limits.)

- Make the limit of the quotient

$$\frac{d(t + h) - d(t)}{h}$$

precise. (This will give rise to the concept of a derivative.)

- The process of computing a derivative is called differentiation. The opposite process is integration. The result of integration is an integral.

- Find formulas for computing derivatives and integrals.

- see lots and lots of applications.

- Math 2210 (Calc III) covers the Calculus of several (dependent or independent variables.)

- This semester we will follow the same schedule as most other 1210 sections. The Labs (mostly) will all have the same activities.