Math 1210-23 Notes of 1/9/24

Reminders

- Survey due today if possible
- Assessment test due on Thursday
- hw 1 is open, due 1/22
- yesterday's recording on Canvas
- Today's annotated notes and recording will be on Canvas some time this afternoon.

What is Calculus?

And why do we need to study it?

- Quick answer to the second question: Because it can be used to solve many problems!

The Key Ideas

- We will introduce the concepts with an example:

Velocity — Location

- Your car has a **speedometer** (showing current velocity) and an **odometer** (showing distance covered, or location along a highway).
- Speed versus velocity: velocity has a direction, in this semester just forward and backward, or up and down, distinguished by a plus or minus sign.
- **Profound Fact:** Velocity and Location are **related**. One determines the other
- Think of both as functions of time t:
  - $v(t)$ is **v**elocity at time t
  - $d(t)$ is location (**d**istance) at time t
- If we know our location function d we should be able to compute our velocity function v,

and if we know our velocity function v (and our location at time $t = 0$, say) we should be able to compute our location function.

- Note: we do not just compute location or velocity at a specific time. We know one function for all (relevant) time and compute the other for all (relevant) time.
- This is what Calculus is all about!
  - $v \longrightarrow d$: integration
  - $d \longrightarrow v$: differentiation

$v(t) = 32t$ ft/sec

$d(t) =$?

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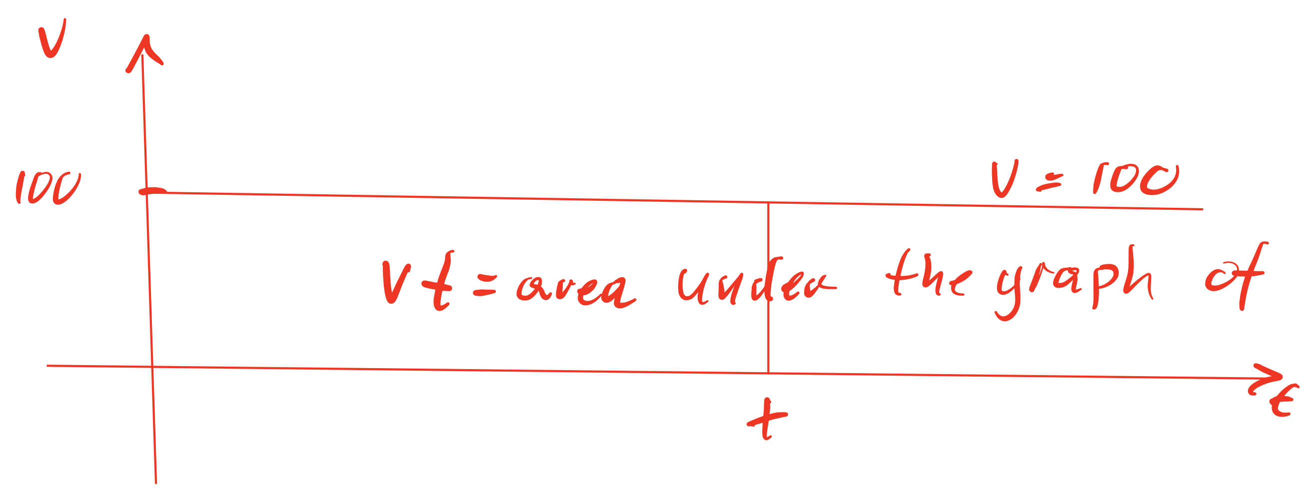
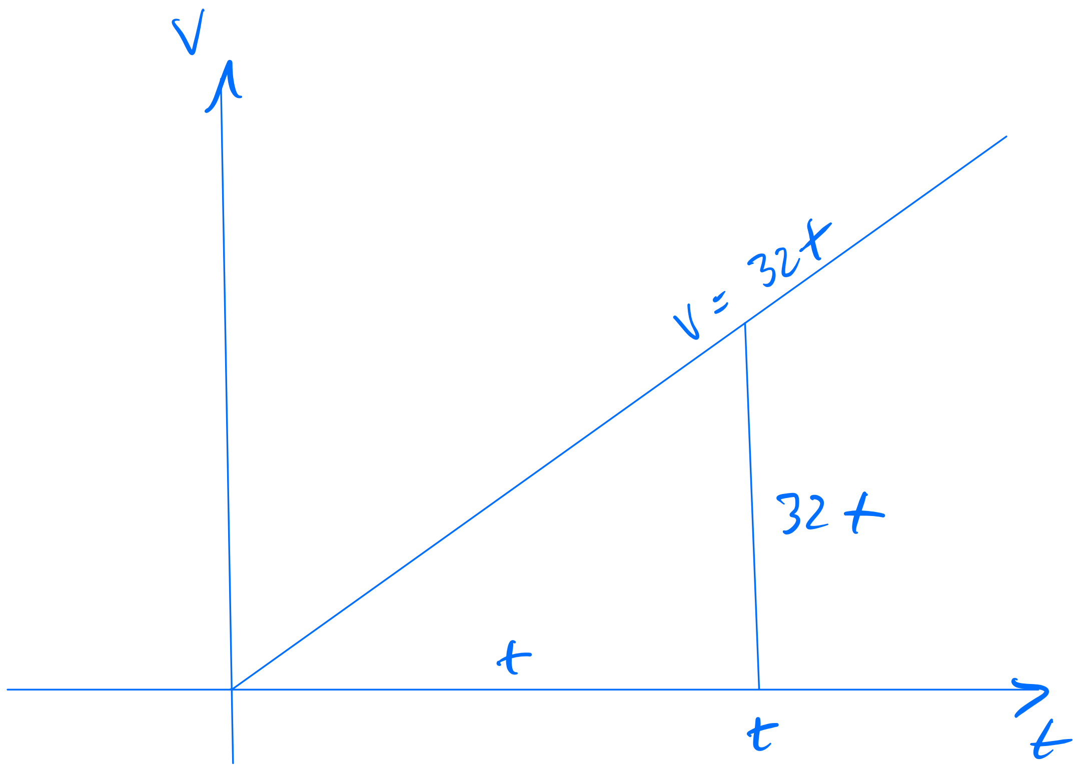
simpler: $v(t) = 100$ mph t # hours

$d(t) = 100t$





$$d(t) = \frac{1}{2}\, 32t \cdot t = 16t^2$$

Falling Objects

- simple physical example: falling object, ignore air resistance, consider gravity constant. (It actually does depend on altitude and location on earth.)
- Observation: Velocity increases by 32 ft/sec every second. We say that the acceleration is 32 feet per second squared.
- Let $v(t)$ be the downward velocity, and assume
$$v(0) = 0.$$
- Then clearly
$$v(t) = 32t$$
- **How far does the object fall in t seconds?**
- In other words, what is $d(t)$? (Think of d as **d**istance or **d**epth.)
- Let's figure it out.

Going the other way

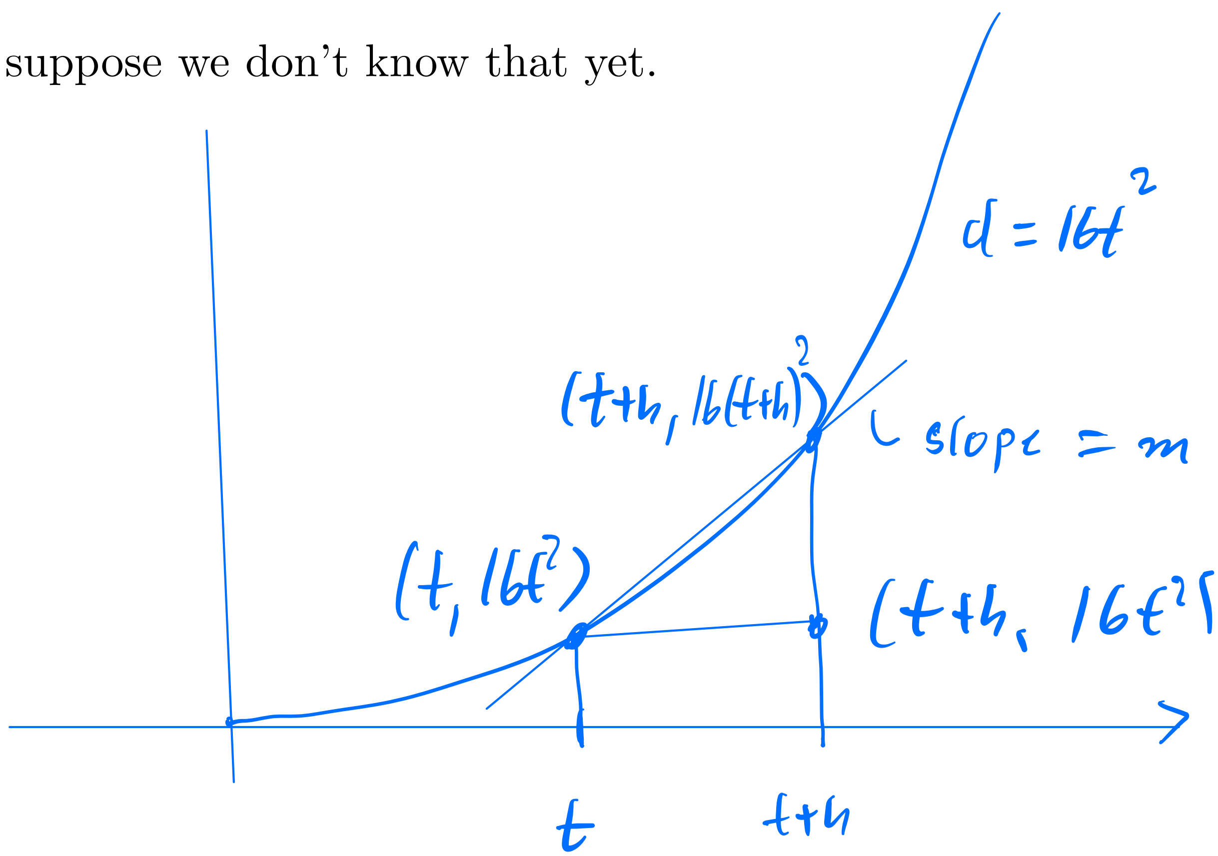
- Suppose we know the distance. How can we figure out the velocity?
- Let's apply the ideas to a situation where we already know the answer!
- Suppose

$$d(t) = 16t^2$$

- What is $v(t)$. Of course we should get

$$v(t) = 32t$$

but suppose we don't know that yet.



 $(t+h)^2 \neq t^2+h^2$

$$m = \frac{16(t+h)^2 - 16t^2}{(t+h) - t}$$

$$= \frac{16(t+h)^2 - 16t^2}{h} \longrightarrow \frac{0}{0}$$

$$= \frac{16(t^2+2th+h^2) - 16t^2}{h}$$

$$(t+h)^2 = (t+h)(t+h)$$

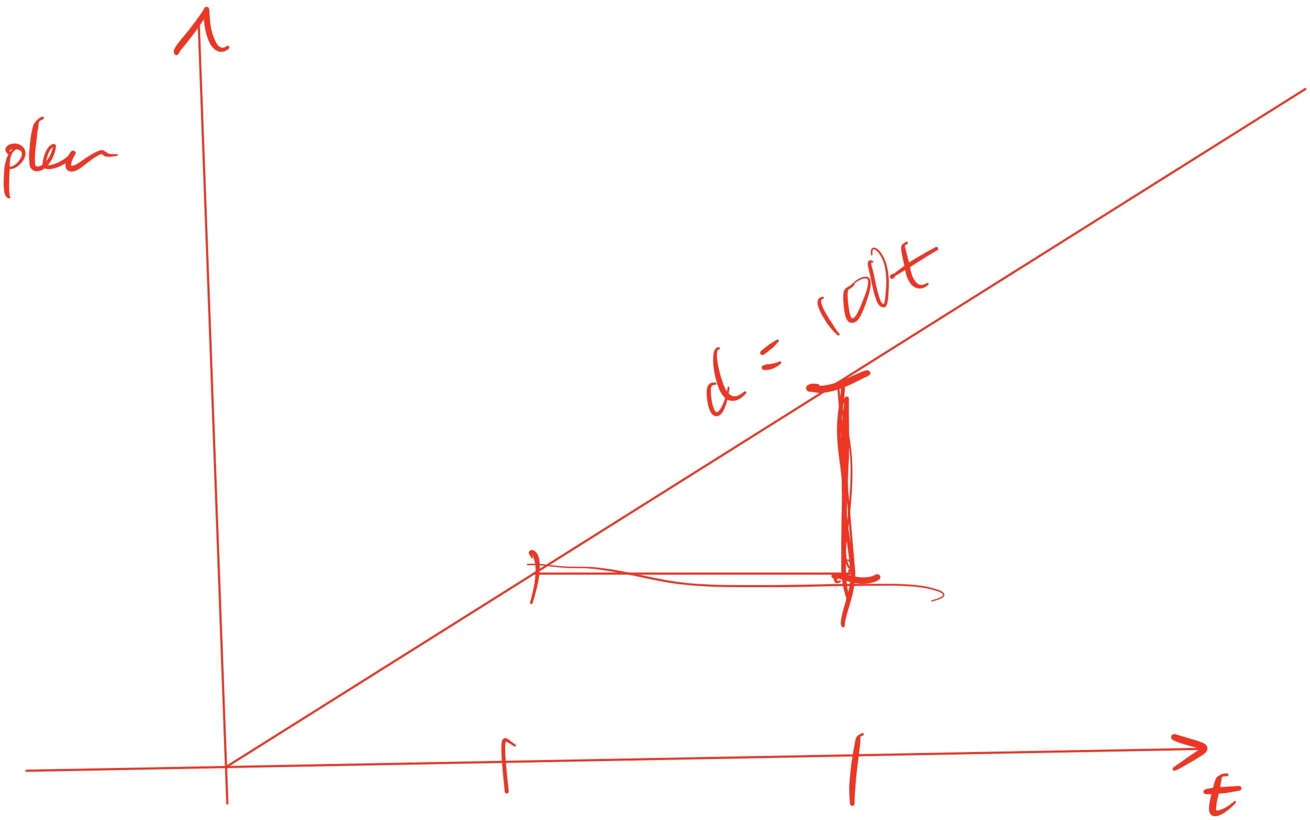
$$= \frac{16t^2 + 32th + 16h^2 - 16t^2}{h}$$

$$= \frac{32th + 16h^2}{h}$$

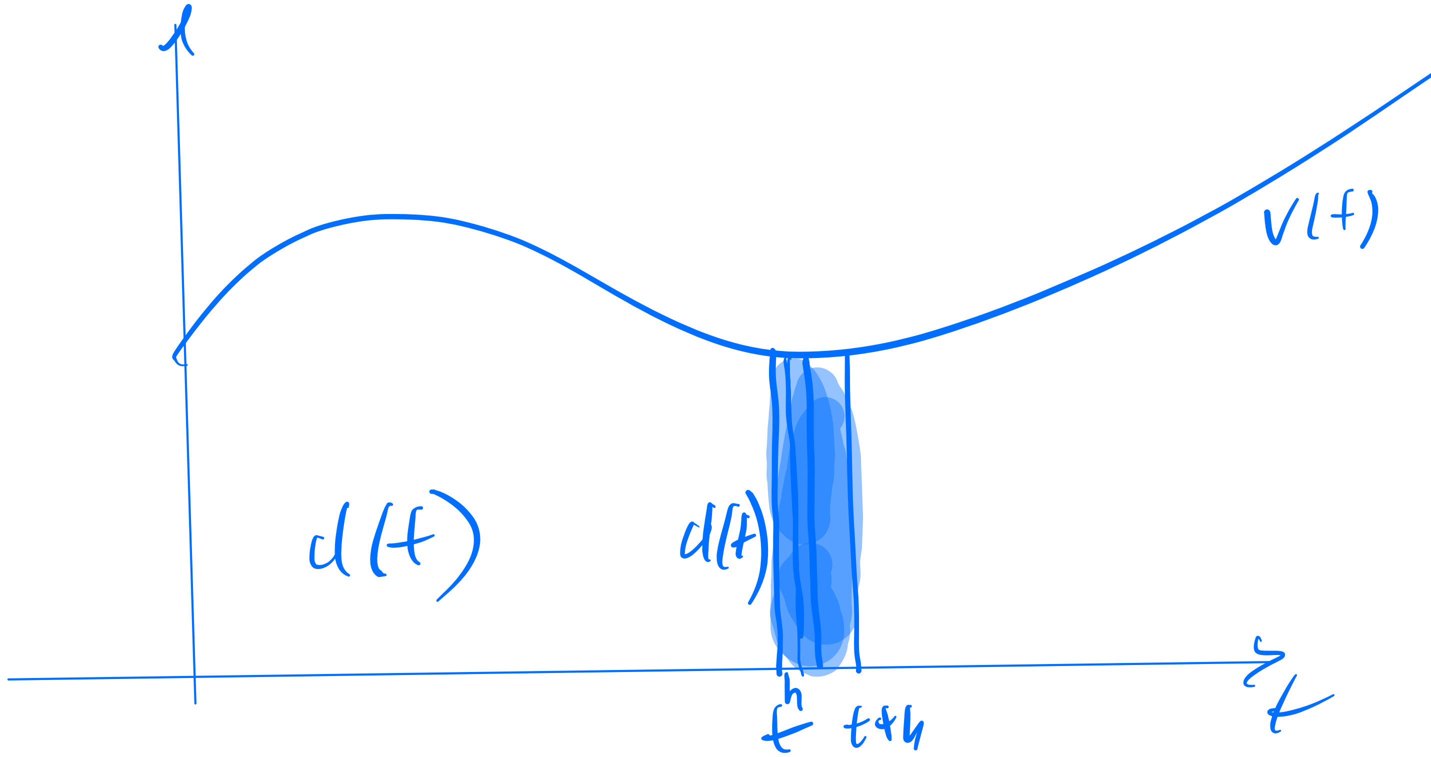
$$= 32t + 16h \longrightarrow 32t$$

as $h \to 0$

simpler



The Fundamental Theorem of Calculus



$$\frac{d(t+h) - d(t)}{h}$$

The Plan (for both Math 1210 and 1220)

- Make the idea $h \longrightarrow 0$ precise. (This will give rise to the concept of **limits**.)
- Make the limit of the quotient

  $$\frac{d(t+h) - d(t)}{h}$$

  precise. (This will give rise to the concept of a **derivative**.)
- The process of computing a derivative is called **differentiation**. The opposite process is **integration**. The result of integration is an **integral**.
- Find formulas for computing derivatives and integrals.
- see lots and lots of applications.
- Math 2210 (Calc III) covers the Calculus of several (dependent or independent variables.)
- This semester we will follow the same schedule as most other 1210 sections. The Labs (mostly) will all have the same activities.