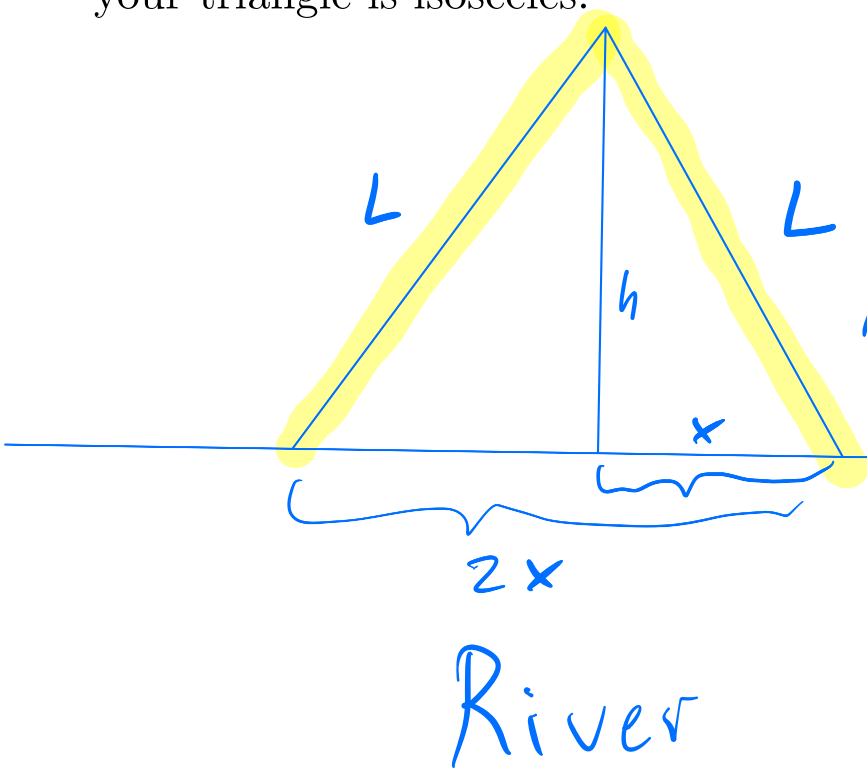
Math 1210-23

Notes of 2/26/24

- Exam 2 on Friday. It will be on Chpt 2 of the textbook, hws 4–7, notes of 1/24–~~16~~. 2/16
- While I am not trying to suggest anybody should withdraw from the class, you may want to know that this coming Friday, March 1, is the last day you can do so without going through an uncertain and complex permission procedure.

3.4 More Extreme Value Problems

- You want to fence a triangular piece of land along a river with $2L$ feet of fencing. You don't need a fence along the river. How do you maximize the fenced in area? Assume your triangle is isosceles.



$x = ?$

$' = \frac{d}{dx}$

$0 \le x \le L$

$2x = L$

$2x < L$

$2x > L$ ✓

$A = \frac{1}{2} h \, 2x = hx = \max$

$h^2 + x^2 = L^2$

$h^2 = L^2 - x^2$

$h = \sqrt{L^2 - x^2}$

$$A = x\sqrt{L^2 - x^2} = \max \qquad (uv)' = u'v + uv'$$

$$A' = \sqrt{L^2 - x^2} + \frac{x \cdot (-2x)}{2\sqrt{L^2 - x^2}} = 0$$

$$0 = \sqrt{L^2 - x^2} - \frac{x^2}{\sqrt{L^2 - x^2}}$$

$$\frac{L^2 - x^2 - x^2}{\sqrt{L^2 - x^2}} = 0$$

$$L^2 - 2x^2 = 0 \Rightarrow L^2 = 2x^2$$

$$x^2 = \frac{L^2}{2}$$

$$\boxed{x = \frac{1}{\sqrt{2}} L} \qquad 2 \cdot \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

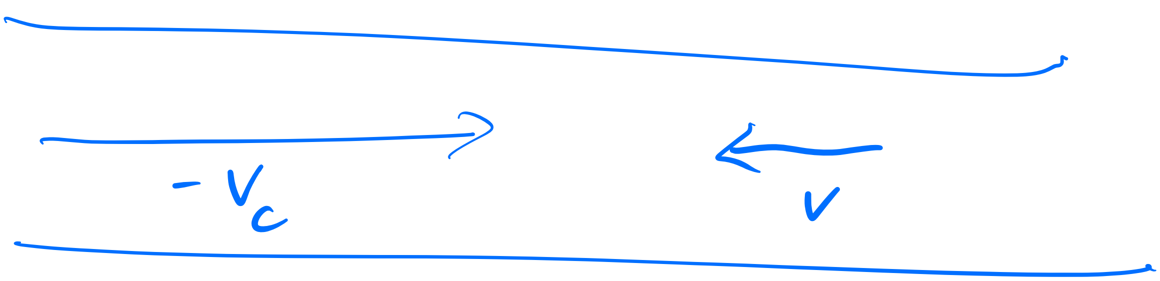
$$2x = \sqrt{2}\, L > L$$

$$x^2 = \sqrt{\frac{L^2}{2}}$$

$$x = \frac{L}{\sqrt{2}}$$

$$2(kL) = (2k)L$$

- Example 4: A fish swims upstream with velocity v relative to the water. The current of the river is $-v_c$. The energy expended in traveling a distance d up the river is directly proportional to the time required to travel the distance and the cube of the velocity. What velocity minimizes the energy?
- Query: What about swimming downstream?
- If you don't care about fish swimming against the current, think of rowing a boat going upstream.



$$E = k v^3 \cdot T \qquad T = \frac{d}{v - v_c}$$

$$\left(\frac{N}{D}\right)' = \frac{N'D - ND'}{D^2}$$

$$E = \underbrace{kd}_{\text{const.}} \frac{v^3}{v - v_c} = \min$$

$$' = \frac{d}{dv}$$

$$E' = kd\,\frac{3v^2(v - v_c) - v^3}{(v - v_c)^2} = 0$$

$$0 = 3v^2(v - v_c) - v^3$$

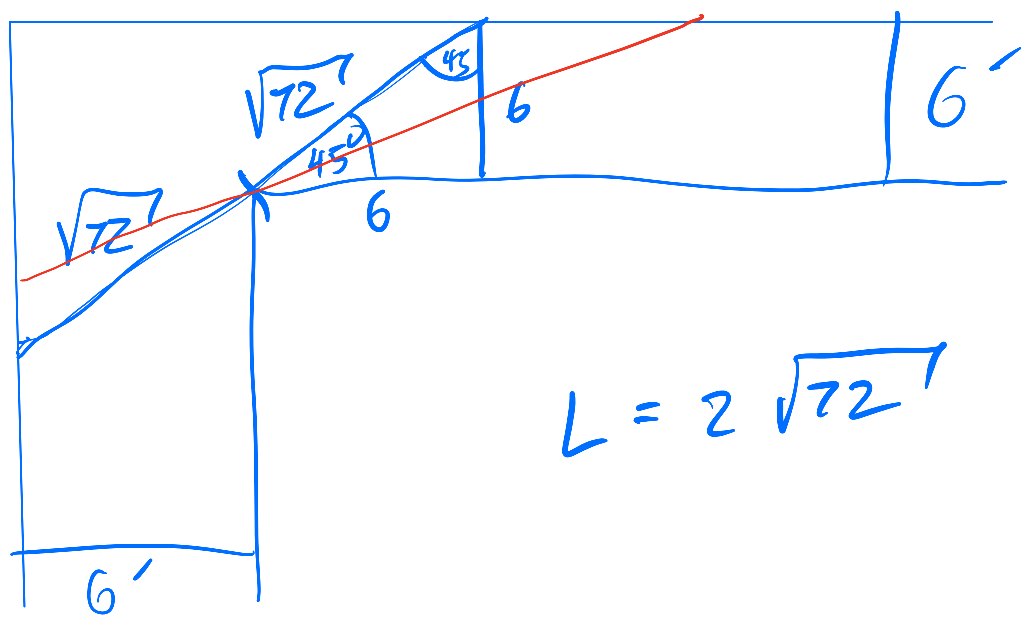
$$= 3v^3 - v^3 - 3v^2 v_c = 0$$

$$2v^3 - 3v^2 v_c = 0 \quad \div v^2$$

$$2v = 3v_c$$

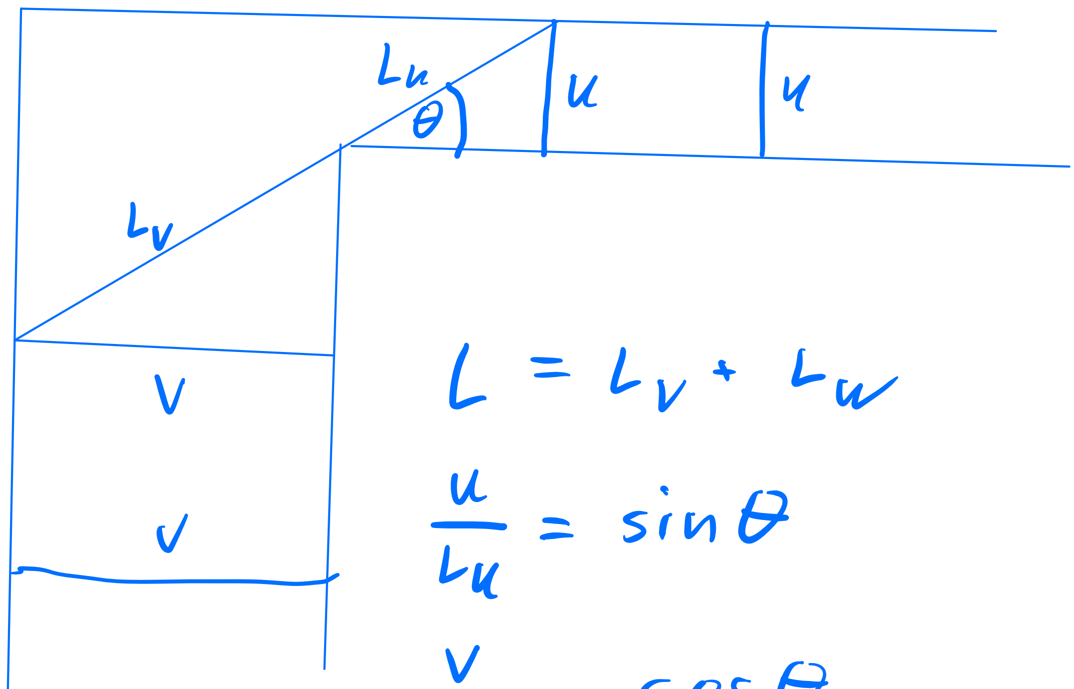
$$v = \frac{3}{2} v_c$$

- Example 5. A 6-foot hallway makes a right turn. What is the the length of the longest thin rod that can be carried around the corner, assuming you cannot tilt the rod. (Think of a sheet of plywood or glass).
- Expectations



$L = 2\sqrt{72}$

- OK, that was easy. But now supposes that the hall changes width, say from u feet to v feet.



$$L = L_V + L_w$$

$$\frac{u}{L_u} = \sin\theta$$

$$\frac{V}{L_V} = \cos\theta$$

$$L_u = \frac{u}{\sin\theta}$$

$$L_V = \frac{V}{\cos V}$$

$$f(\theta) = L_u + L_V = \frac{u}{\sin\theta} + \frac{V}{\cos\theta} = \min$$

$$f'(\theta) = \frac{-u\cos\theta}{\sin^2\theta} + \frac{+V\sin\theta}{\cos^2\theta} = 0$$

$$1 \cdot \sin^2\theta\cos^2\theta$$

$$-u\cos^3\theta + v\sin^3\theta = 0$$

$$u\cos^3\theta = v\sin^3\theta$$

$$u = v\,\frac{\sin^3\theta}{\cos^3\theta}$$

$$\frac{u}{v} = \frac{\sin^3\theta}{\cos^3\theta} = \tan^3\theta$$

$$\tan\theta = \left(\frac{u}{v}\right)^{1/3}$$

$$\theta = \arctan\left(\frac{u}{v}\right)^{1/3}$$

$$\frac{d}{d\theta}\frac{1}{\sin\theta} = \frac{\qquad}{\sin^2\theta}$$

- If time permits: You want to build a square box holding a volume V. The material for the top is 10 times as expensive as the material for the bottom and the four sides. Let h be the height of the top and s the length of a side. What is the ratio h/s that minimizes the cost?