Math 1210-23

Notes of 02/20/24

- We are done with Chapter 2. Exam 2 will cover Chapter 2, and will take place next week, after the last hw covering chapter 2 is closed.

3.1 Minima and Maxima

- Entering new chapter
- Minimization and Maximization: obviously important, and major application of Calculus.
- Example: Recall an old home work problem: What's the maximum value of

$$f(x) = x^2(1 - x)$$

in the interval $[0, 1]$?

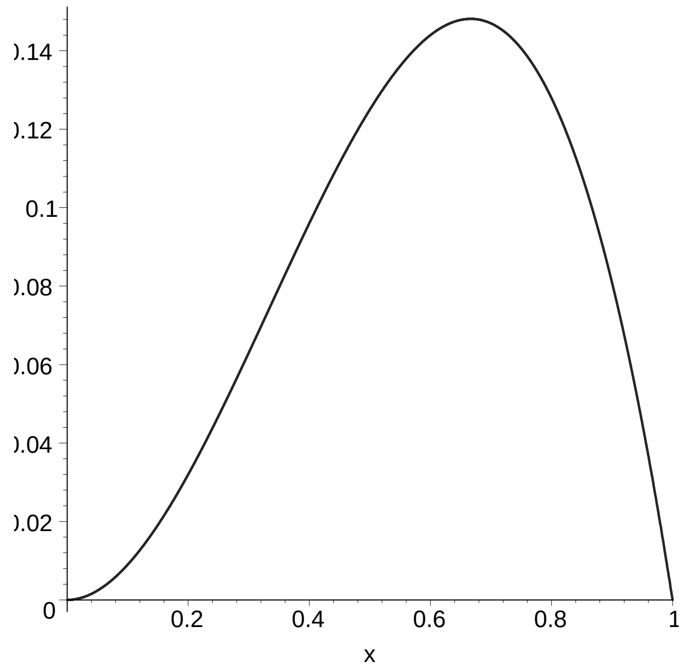

Figure 1. Graph of $f(x) = x^2(1 - x)$.

- Example:

$$f(x) = \sin x$$

- Maxima at

$$x = 2n\pi + \frac{\pi}{2} \qquad n \text{ integer},$$

- Minima at

$$x = 2n\pi + \frac{3\pi}{2} \qquad n \text{ integer}.$$

We have $f'(x) = \cos x = 0$ at those points.

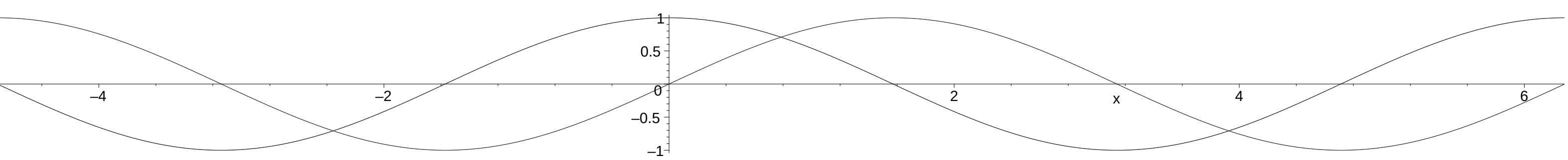

Figure 2. Sine and Cosine.

Vocabulary

- Suppose $y = f(x)$, f a given function.
- f has a **maximum value** $f(c)$ at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f.
- f has a **minimum value** $f(c)$ at $x = c$ if $f(c) \leq f(x)$ for all x in the domain of f.
- f has an **extreme value** $f(c)$ at $x = c$ if it has a minimum or maximum value at $x = c$.
- We need to refine this terminology later.
- So at what kind of points c can we get extreme values?
- It turns out that there are just three kinds:
- c is a **stationary point** if $f'(c) = 0$.
- c is a **singular point** if $f'(c)$ does not exist.
- c is a **boundary point** if is a boundary point of the domain of f.
- Collectively, these three kinds of points are called **critical points**.
- Extreme values can occur only at critical points!
- The word **point** refers to a value of the independent variable, not to a point in the plane.

- why only those three kinds of points?

- Example:

$$f(x) = -2x^3 + 3x^2 \qquad\qquad -\frac{1}{2} \le x \le 2$$

- The calculations are consistent with the graph:

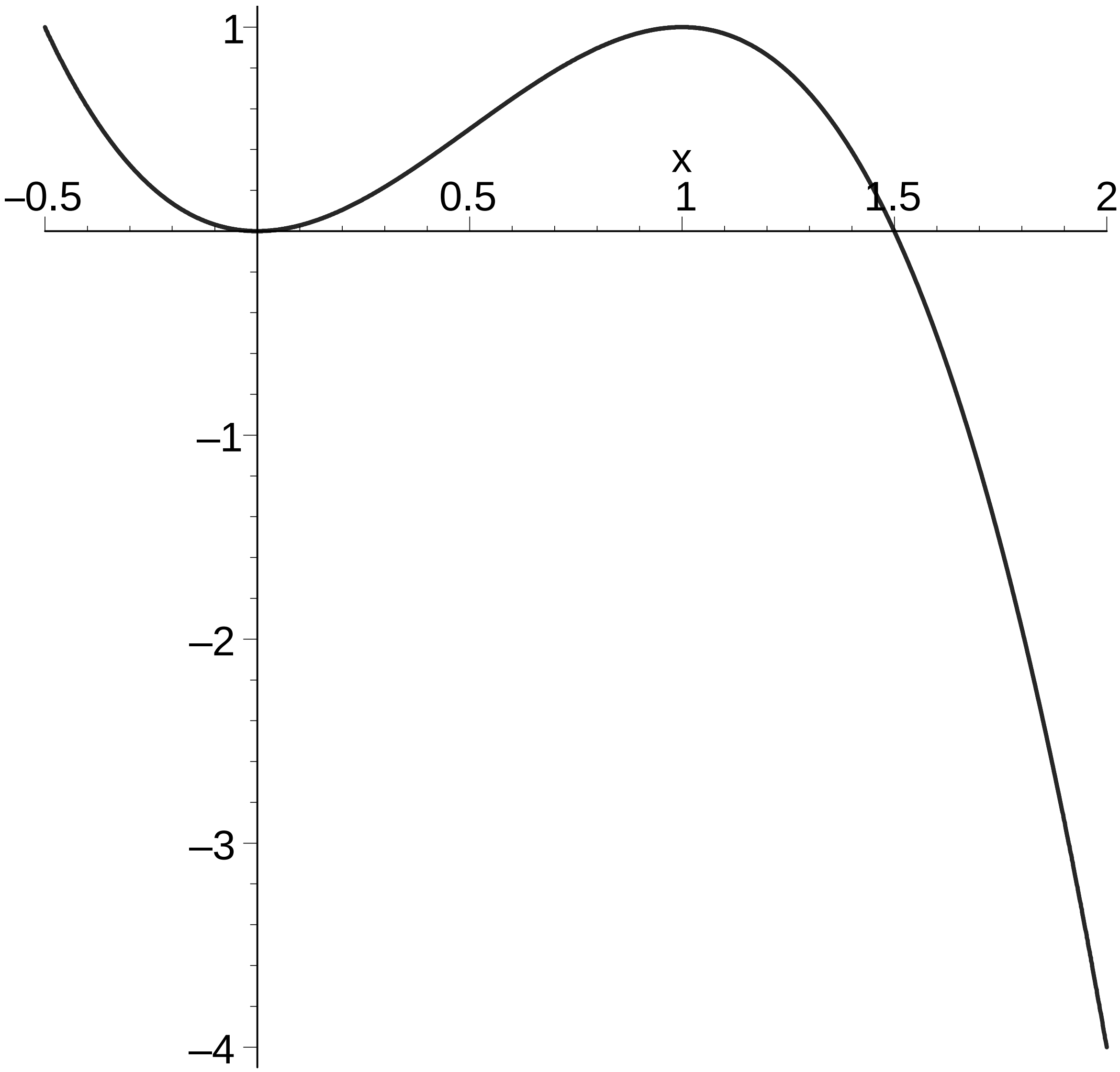

Figure 3. Graph of $f(x) = -2x^3 + 3x^2$, $-\frac{1}{2} \leq x \leq 2$.

- Example 2: $f(x) = x^3$, $-2 \le x \le 2$.

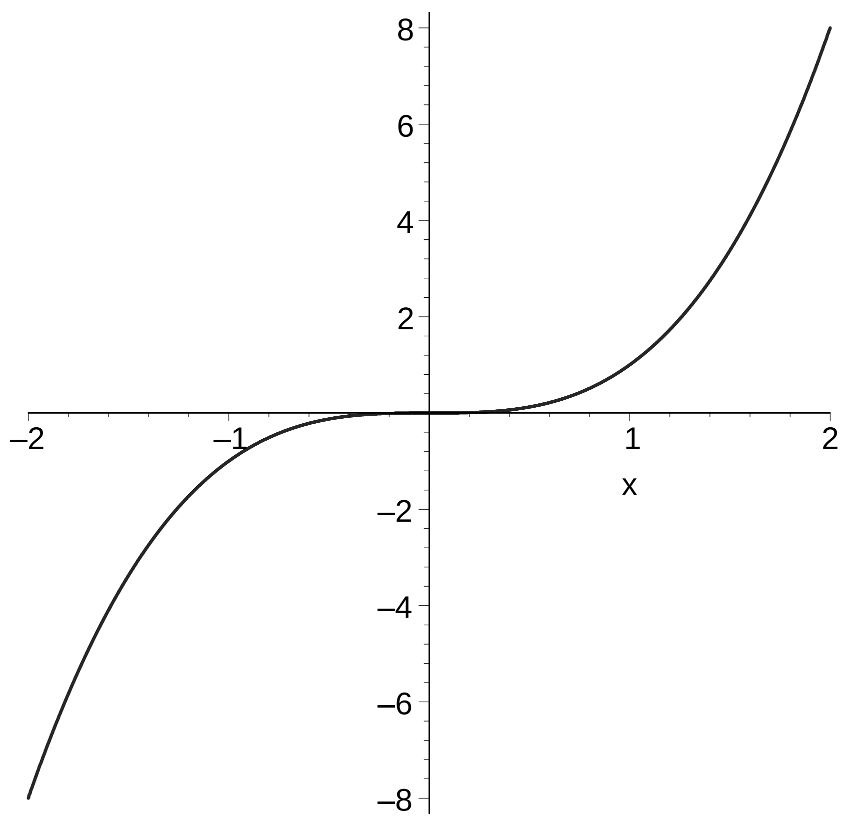

Figure 4. Graph of $f(x) = x^3$.

General Procedure

- So, to find extreme values of f, we proceed as follows:

1. Find all critical points (stationary, singular, or boundary) of f.

2. Evaluate f at those critical points.
3. Find the min and the max.

- Often there are no singular points, and in word problems what happens at the boundary points is often obvious.

- Example: Find the extreme values of

$$f(x) = \frac{x}{1+x^2}$$

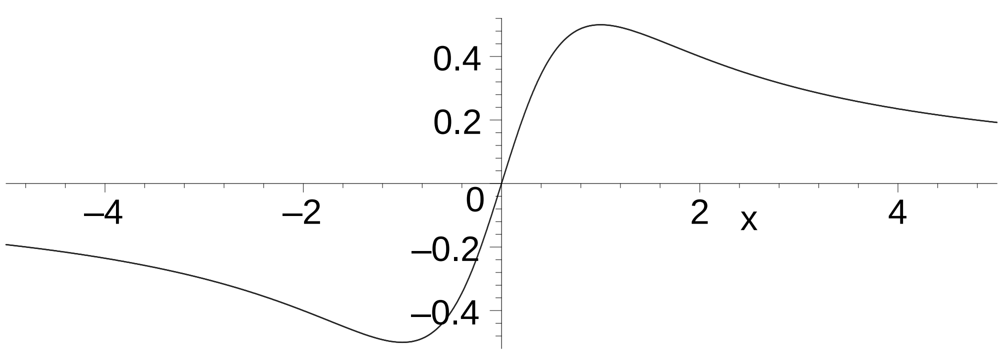

Figure 5. Graph of $f(x) = \frac{x}{1+x^2}$.

- A word problem: You design the layout of a (large, fancy, coffee-table) book. Each page should have an area of 72 square inches for text and pictures, 1 inch wide margins on the left and right, and 2 inch margins on top and bottom. What are the dimensions of each page that minimize the overall area (and cost) of each page?
- First think of expectations. Is the page going to be square, landscape (wider than high), or portrait (higher than wide)?
- How much higher? Twice as high, more than twice, less than twice?