We are done with Chapter 2. Exam 2 will cover Chapter 2, and will take place next week, after the last hw covering chapter 2 is closed.

### 3.1 Minima and Maxima

- Entering new chapter
- Minimization and Maximization: obviously important, and major application of Calculus.
- Example: Recall an old home work problem: What’s the maximum value of

\[ f(x) = x^2(1 - x) \]

in the interval \([0, 1]\)?
Figure 1. Graph of $f(x) = x^2(1 - x)$. 
• Example:
  \[ f(x) = \sin x \]

• Maxima at
  \[ x = 2n\pi + \frac{\pi}{2} \quad n \text{ integer}, \]

• Minima at
  \[ x = 2n\pi + \frac{3\pi}{2} \quad n \text{ integer}. \]

We have \( f'(x) = \cos x = 0 \) at those points.

\[ \text{Figure 2. Sine and Cosine.} \]
Vocabulary

- Suppose \( y = f(x) \), \( f \) a given function.
- \( f \) has a **maximum value** \( f(c) \) at \( x = c \) if 
  \[ f(c) \geq f(x) \] 
  for all \( x \) in the domain of \( f \).
- \( f \) has a **minimum value** \( f(c) \) at \( x = c \) if 
  \[ f(c) \leq f(x) \] 
  for all \( x \) in the domain of \( f \).
- \( f \) has an **extreme value** \( f(c) \) at \( x = c \) if it 
  has a minimum or maximum value at \( x = c \).
- We need to refine this terminology later.
- So at what kind of points \( c \) can we get extreme values?
- It turns out that there are just three kinds:
  - \( c \) is a **stationary point** if \( f'(c) = 0 \).
  - \( c \) is a **singular point** if \( f'(c) \) does not exist.
  - \( c \) is a **boundary point** if \( c \) is a boundary point 
    of the domain of \( f \).
- Collectively, these three kinds of points are 
  called **critical points**.
- Extreme values can occur only at critical points!
- The word **point** refers to a value of the independent variable, 
  not to a point in the plane.
• why only those three kinds of points?
• Example:

\[ f(x) = -2x^3 + 3x^2 \quad \frac{1}{2} \leq x \leq 2 \]
• The calculations are consistent with the graph:

Figure 3. Graph of $f(x) = -2x^3 + 3x^2$, $-\frac{1}{2} \leq x \leq 2$. 
• Example 2: $f(x) = x^3$, $-2 \leq x \leq 2$.

Figure 4. Graph of $f(x) = x^3$. 
General Procedure

- So, to find extreme values of $f$, we proceed as follows:

1. Find all critical points (stationary, singular, or boundary) of $f$.
2. Evaluate $f$ at those critical points.
3. Find the min and the max.

- Often there are no singular points, and in word problems what happens at the boundary points is often obvious.
Example: Find the extreme values of

\[ f(x) = \frac{x}{1 + x^2} \]

Figure 5. Graph of \( f(x) = \frac{x}{1 + x^2} \).
A word problem: You design the layout of a book. Each page should have an area of 72 square inches for text, 1 inch wide margins on the left and right, and 2 inch margins on top of bottom. What are the dimensions of each page that minimize the overall area?