Math 1210-23

Notes of 2/14/22

2.8 More on Related Rates

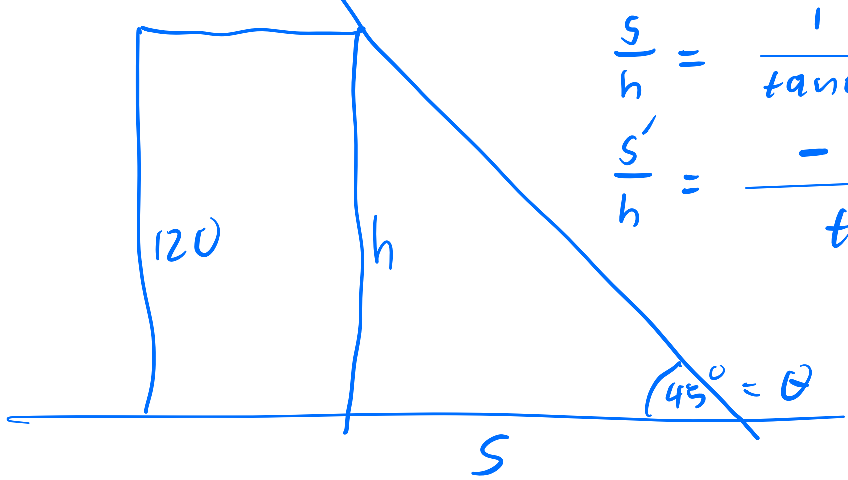
- As a warm up, let's look at an example (Example 5, textbook).
- As the sun sets behind a 120 feet tall building the building's shadow grows. How fast is the shadow growing when the sun's rays make an angle θ of 45 degrees with the horizontal?
- The textbook tacitly assumes that θ changes by 2π in 86,400 seconds. This is true on the equator on the first day of spring or fall, but not in general.

$$\tan\theta = \frac{h}{s}$$

$$\frac{s}{h} = \frac{1}{\tan\theta}$$

$$\frac{s'}{h} = \frac{-\frac{\theta'}{\cos^2\theta}}{\tan^2\theta} = -\frac{\theta'}{\cos^2\theta\,\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= -\frac{\theta'}{\sin^2\theta}$$



$$s' = -\frac{h\theta'}{\sin^2\theta}$$

(recording has an error in this calculation)

Summary of RR technique

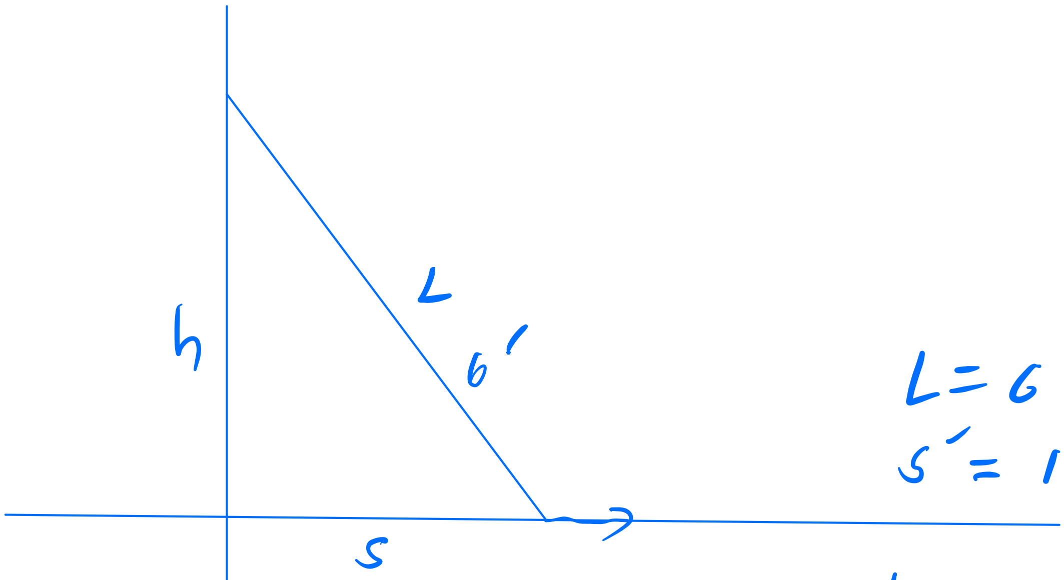
1. Assign variables to quantities that depend on time.

   I strongly recommend that you also assign variables to all numerically given data. You get a more general solution, and you can increase your confidence in your calculation by checking the dimensions of your mathematical expressions.

2. Write one or more equations relating the variables, and encapsulating what is known about the problem.
3. Differentiate with respect to time in those equations, keeping clearly in mind which variables do depend on time, and which do not.
4. Solve for required variables or their derivatives.
5. Substitute specific values for the variables.

You must have variables for all quantities that change with time when you differentiate. If you substitute constants for variables before differentiating you get 0=0.

- Example: You are pulling the bottom of a six foot ladder away from a wall at a constant speed of 1 foot per second.
- At what speed does the top of the ladder hit the ground?



$L = 6$

$s' = 1$ f/s

$h = 0$ $\quad s = 6$

$$s^2 + h^2 = L^2$$

$$2ss' + 2hh' = 0$$

$$ss' = -hh'$$

$$h' = -\frac{ss'}{h} = -\frac{6 \cdot 1}{0} \longrightarrow -\infty$$

- Example: A (spherical) watermelon's radius is growing at one inch per week. How fast is its volume increasing in general?

$$V = \frac{4}{3}\pi r^3$$

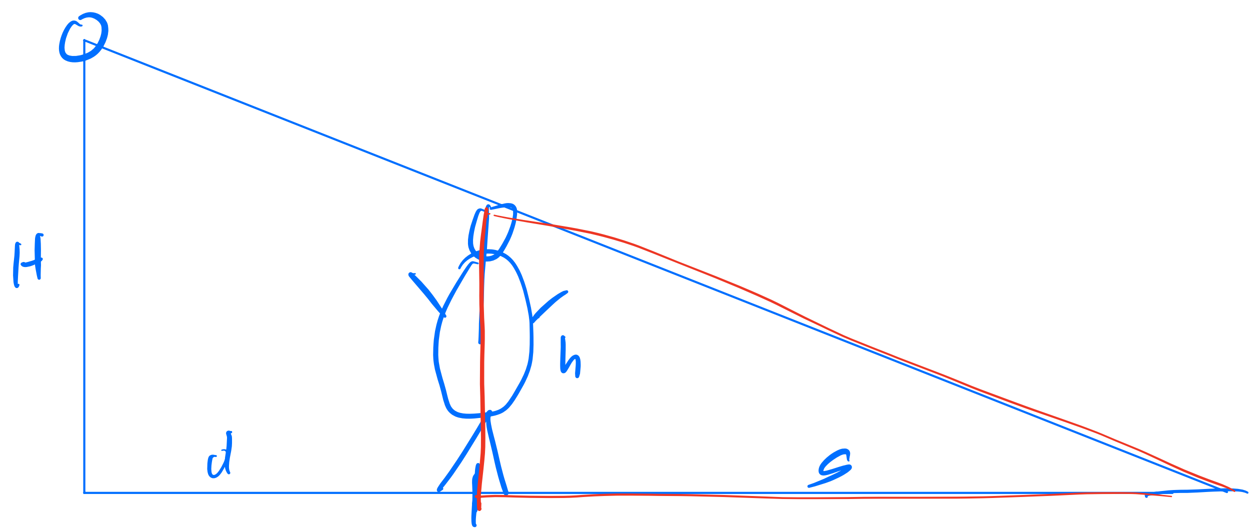
$$\left(A = 4\pi r^2\right)$$

$$V' = \frac{4}{3}\pi 3r^2 r' = V' = 4\pi r^2 r'$$

$$= 4\pi r^2 \cdot 1$$

- Example: You are six feet tall and you are walking away at a constant speed of 2 feet per second from a street light that's 10 feet tall. How fast is your shadow growing when you are 20, 30, 40, 4000 feet away from the lamp post? How fast is the tip of your shadow moving away from the wall? What are your expectations?

5f/s



$$\frac{s}{h} = \frac{s+d}{H}$$

$$\frac{s'}{h} = \frac{s'+d'}{H}$$

$$s'\left(\frac{1}{h} - \frac{1}{H}\right) = \frac{d'}{H}$$

$$s' = \frac{d'}{H\left(\frac{1}{h} - \frac{1}{H}\right)} \cdot \frac{hH}{hH}$$

$$s' = \frac{d' h H}{H(H-h)}$$

$$= \frac{d' h}{H}$$

$d' = 2 \quad h = 6$

$H - h = 4$

$$s' = \frac{2 \cdot 6}{4}$$

$$= 3 \quad f/s$$

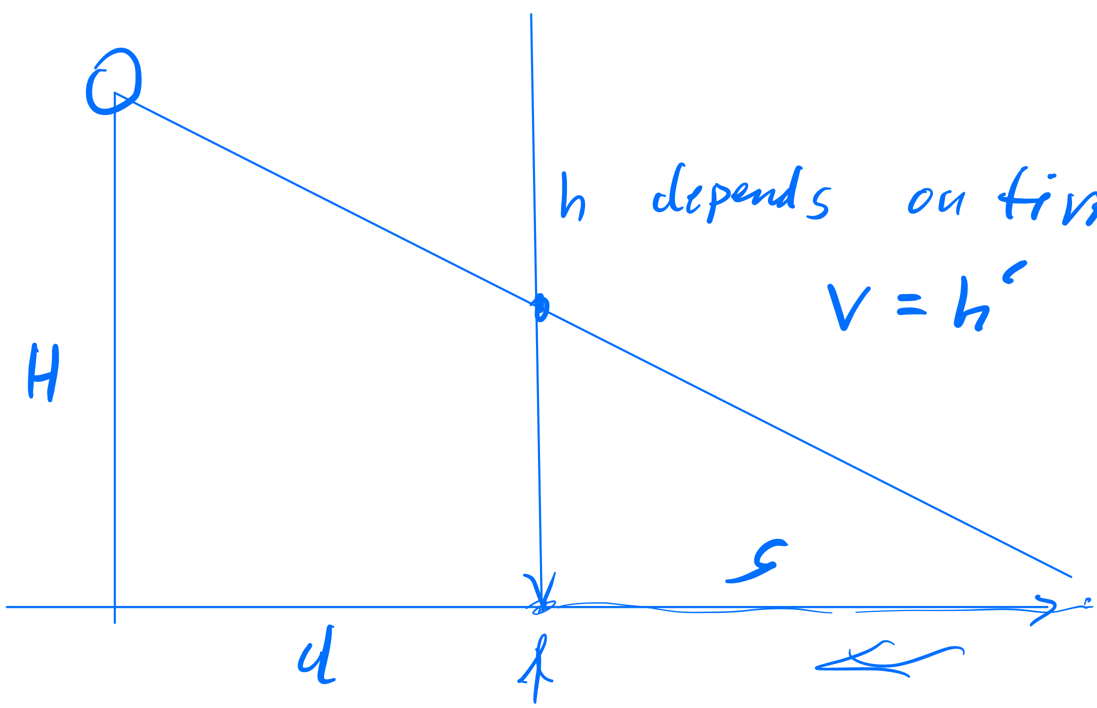
$$s'\left(\frac{1}{h} - \frac{1}{H}\right) = \frac{d'}{H}$$

$$\frac{1}{h} - \frac{1}{H} = \frac{H - h}{hH}$$

$$s' = \frac{\frac{d'}{H} hH}{H-h} = \frac{d' h}{H-h}$$

- ~~Example: You are blowing air into a spherical balloon at the rate of 1 liter per second. (1 liter equals 1000 cubic centimeters.) At what rate is the radius of the balloon increasing when the radius equals 10 centimeters? (If time allows do this explicitly and implicitly.)~~

#2 on hw 7



h depends on time

$V = h'$

$$\frac{s}{h} = \frac{s+d}{H}$$

$$\frac{s'h - sh'}{h^2} = \frac{s'}{H}$$

$$s'\underbrace{\left(\frac{1}{h}-\frac{1}{H}\right)}_{\frac{H-h}{hH}} = \frac{s\,h'}{h^2}$$

$$\frac{s}{h} = \frac{s+d}{H}$$

$$s' = \frac{s h' \, hH}{h^2 (H-h)}$$

$$= \frac{s h' H}{h(H-h)}$$

$$s = h = 0$$

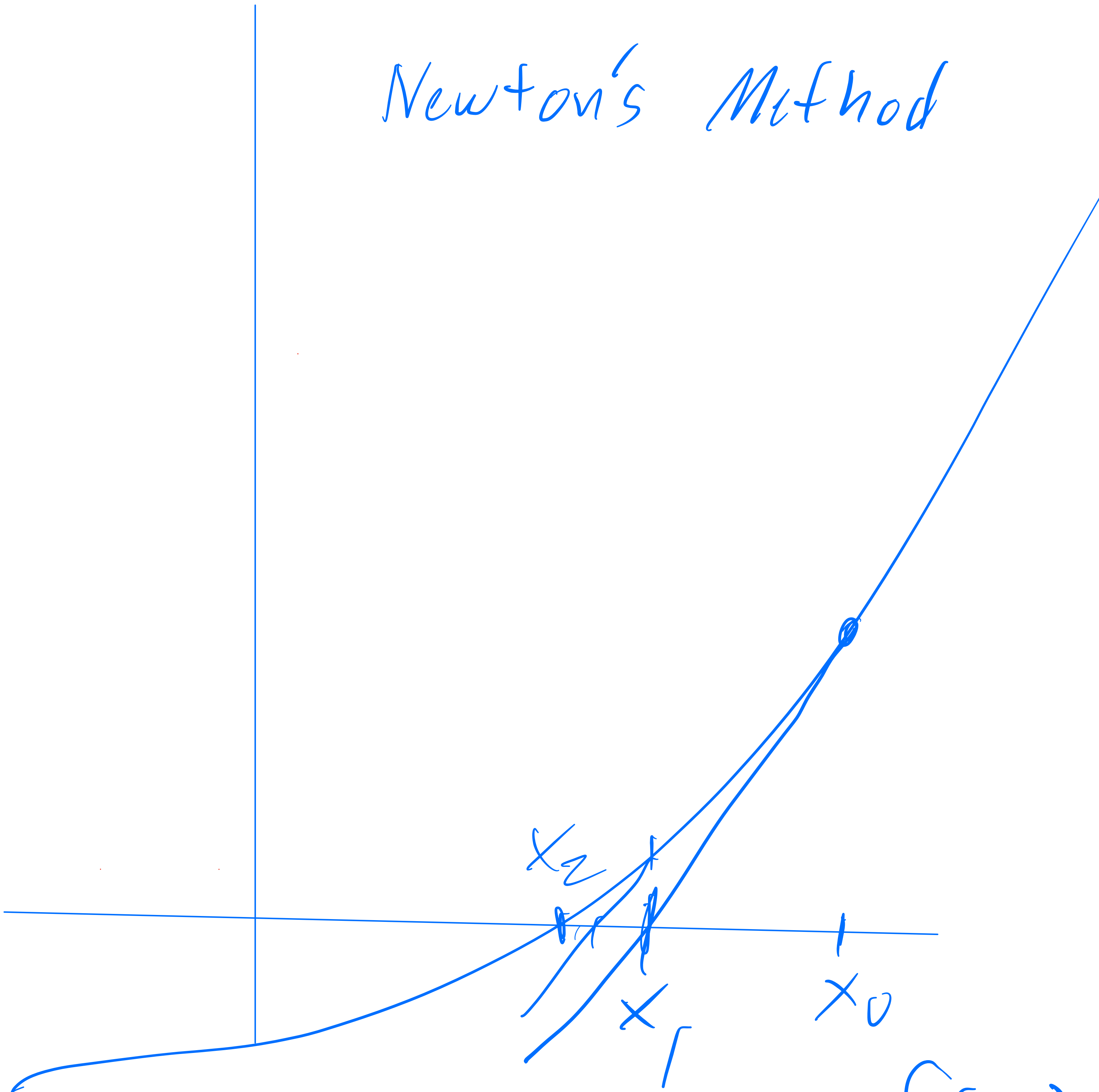
$$= \frac{(s+d)h' \cancel{H}}{\cancel{H}(H-h)}$$

$$s = h = 0$$

$$s' = \frac{d\,h'}{H}$$

$$f(x) = 0$$

Newton's Method



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$