Math 1210-23, Spring 2024

Notes of 2/12/24

- Recall our differentiation rules:

$(kf)'$	$=$	kf'	**Constant Multiple Rule**
$(f+g)'$	$=$	$f'+g'$	**Sum Rule**
$(x^r)'$	$=$	rx^{r-1}	**Power Rule (r rational)**
$(fg)'$	$=$	$f'g+fg'$	**Product Rule**
$\left(\frac{f}{g}\right)'$	$=$	$\frac{f'g-fg'}{g^2}$	**Quotient Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\sin x$	$=$	$\cos x$	**Sine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\cos x$	$=$	$-\sin x$	**Cosine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}f\big(g(x)\big)$	$=$	$f'\big(g(x)\big)g'(x)$	**Chain Rule**

More on Implicit Differentiation

- Section 2.7 continued
- Example 3, textbook. Find the equation of the tangent line to the curve

$$y^3 - xy^2 + \cos xy = 2 \tag{1}$$

at the point $(0, 1)$.

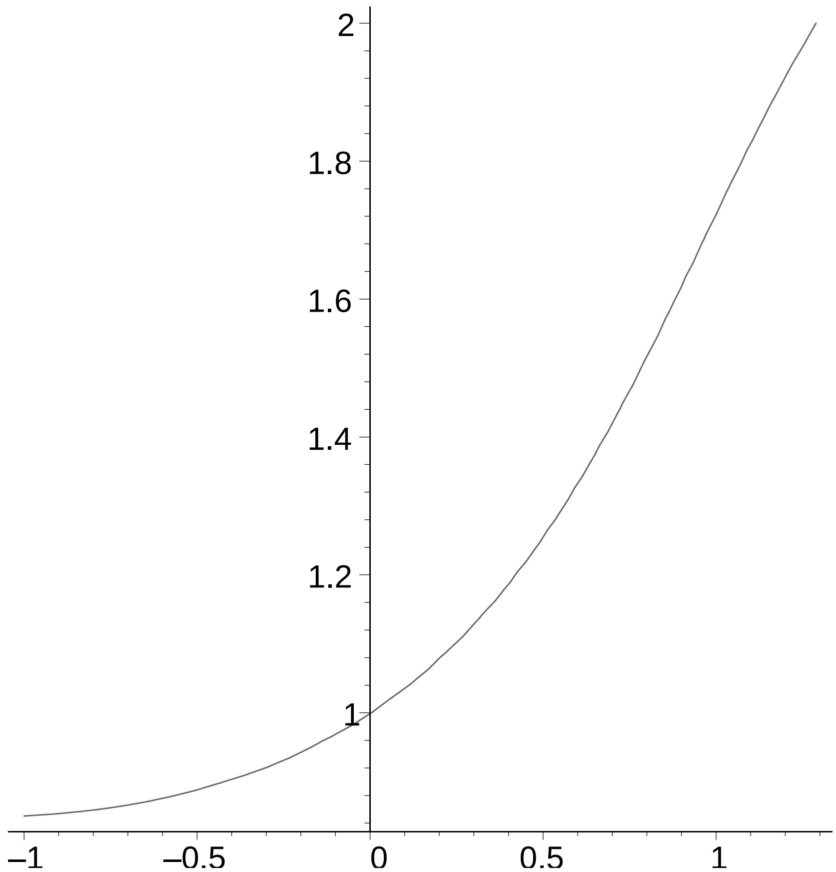

Figure 1. $y^3 - xy^2 + \cos xy = 2.$

- Figure 1 shows the graph of equation (1).

Naturally, implicit differentiation can be done more than once.

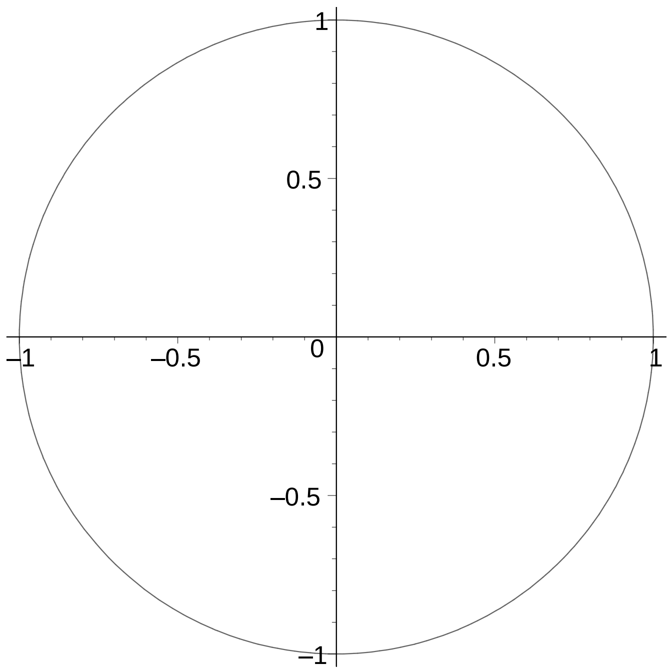

Figure 2. $x^2 + y^2 = 1$.

- Example: Suppose the function $y = f(x)$ is defined by the equation

$$x^2 + y^2 = 1. \qquad (2)$$

  Compute y' and y'' and express it in several ways.

Of course, the graph of (2) is the unit circle, see Figure 2. It does not define a single function, it fails the vertical line test. We can do our stuff only locally, on part of the unit circle.

- Example: Compute y' where $y = f(x)$ is defined by the equation

$$\sin(y) = x$$

and y is in the interval $\left[-\frac{\pi}{2}\,\frac{\pi}{2}\right]$.

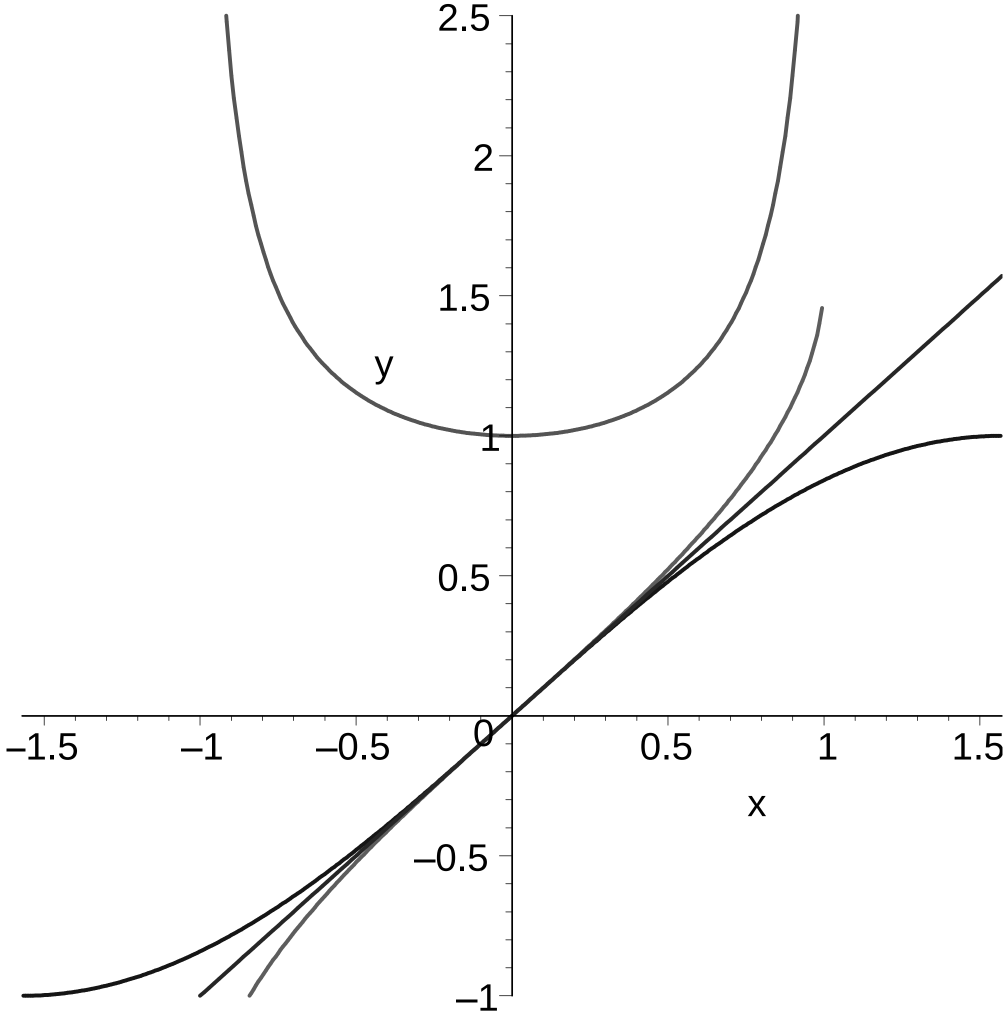

Figure 3. $\sin(y) = x$.

- Problem 43, page 134. Show that the graphs of
$$2x^2 + y^2 = 6 \quad \text{and} \quad y^2 = 4x$$
intersect at right angles.

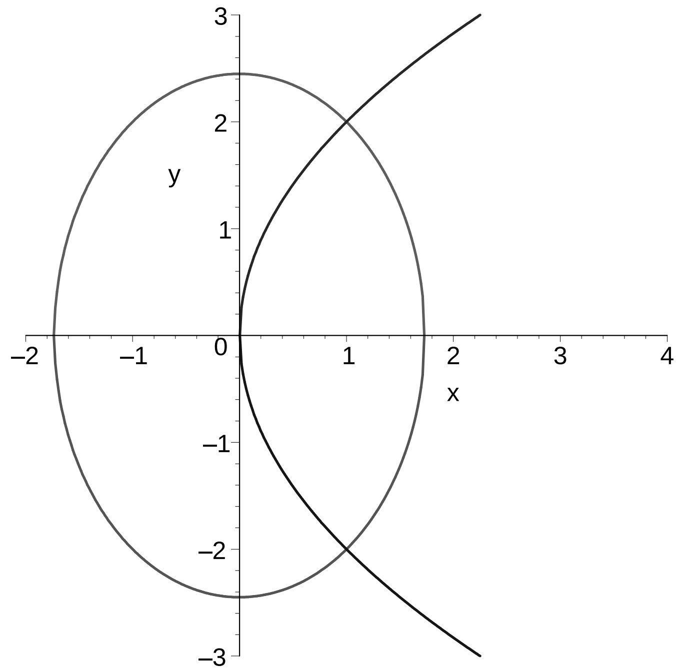

Figure 4. Intersection of $2x^2 + y^2 = 6$ and $y^2 = 4x$.

The intersections are shown in Figure 4.

2.8 Related Rates

- Example: You are blowing air into a balloon at the rate of one cubic foot per minute. How fast is the radius growing when the radius is one foot? Two feet?