Taking Stock

• Suppose $f$ and $g$ are functions of $x$, $k$ is constant, and $n$ is a positive integer.

• So far, we have the following differentiation rules:

\[
(kf)' = kf' \quad \text{Constant Multiple Rule}
\]

\[
(f + g)' = f' + g' \quad \text{Sum Rule}
\]

\[
(x^n)' = nx^{n-1} \quad \text{Power Rule}
\]

\[
(fg)' = f'g + fg' \quad \text{Product Rule}
\]

\[
\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad \text{Quotient Rule}
\]

\[
\frac{d}{dx} \sin x = \cos x \quad \text{Sine Rule}
\]

\[
\frac{d}{dx} \cos x = -\sin x \quad \text{Cosine Rule}
\]

• The first and second property together are equivalent to saying that

\text{Differentiation is Linear}
2.5 The Chain Rule

- Recall function composition

\[ f \circ g(x) = f(g(x)) \].

- The Chain Rule can be written as

\[
\frac{d}{dx} f(g(x)) = f'(g(x))g'(x).
\]

- In other words, the derivative of the composition is the product of the derivatives.

**Examples**

\[
\frac{d}{dx} (x^2 + 1)^{100} =
\]
\[
\frac{d}{dx} \sin^2 x =
\]
\[ \frac{d}{dx} \sin^2 x = \]

\[ \frac{d}{dx} \sin x = \]
• Why does the Chain Rule work?
• Here is another more suggestive way, using Leibniz Notation:

• Let

\[ u = g(x). \]

• Then, by the chain rule

\[
\frac{df}{dx} = \frac{d}{dx} f(g(x)) = f'(u)g'(x) = \frac{df}{du} \frac{du}{dx}.
\]

• The \( du \) “cancel”

• Leibniz notation is often used for this kind of mental crutch.
More Examples

\[ \frac{d}{dx} \sin(\cos(x)) = \]

\[ \frac{d}{dx} \sin(\cos(x^2)) = \]

\[ \frac{d}{dx} \sin(\cos^2(x)) = \]
\[
\frac{d}{dt} \left( \frac{3t-2}{t+5} \right)^2 =
\]

\[
\frac{d}{ds} \left( \frac{s^2-9}{s+4} \right)^2 =
\]
• Recall: a function is **even** if

\[ f(x) = f(-x) \]

for all \( x \) in the domain, and it is **odd** if

\[ f(x) = -f(-x) \]

• Show that the derivative of an even function is odd.

• Similarly, the derivative of an odd function is even.
• More Examples:

\[
\frac{d}{dx} \frac{(x^2+x)^4 - \sin x}{2-\sin x} =
\]

\[
\frac{d}{dx} \sin \frac{1}{x} - \frac{1}{\sin x} =
\]