Math 1210-23

Notes of 2/9/24

• We are heading into some tricky territory and will spend two meetings on section 2.7 of our textbook.

• Recall our differentiation rules:

\[
\begin{align*}
(kf)' &= kf' & \text{Constant Multiple Rule} \\
(f + g)' &= f' + g' & \text{Sum Rule} \\
(x^n)' &= nx^{n-1} & \text{Power Rule (n integer)} \\
(fg)' &= f'g + fg' & \text{Product Rule} \\
\left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2} & \text{Quotient Rule} \\
\frac{d}{dx} \sin x &= \cos x & \text{Sine Rule} \\
\frac{d}{dx} \cos x &= -\sin x & \text{Cosine Rule} \\
\frac{d}{dx} f(g(x)) &= f'(g(x))g'(x) & \text{Chain Rule} \\
\frac{d}{dx} \sqrt{x} &= \frac{1}{2\sqrt{x}} & \text{Square Root Rule}
\end{align*}
\]
2.7 Implicit Differentiation

• As usual, this is only a small extension of what we did previously. But it may still surprise you.

• We’ve been talking about how to differentiate, using our differentiation rules.

• That’s what we’ll continue to do today.

• But we won’t necessarily assume that our function is defined explicitly.

• Example:

\[
\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}
\]
• Example: Do

\[ f(x) = \sqrt{1 - x^2} \]

explicitly and implicitly
• Another example:

\[ y = y(x), \quad y^3 + 7y = x^3 \]

• We think of \( y \) as a function of \( x \)

• It’s easy to check that the point \((2, 1)\) is on the graph.

• What is \( y'(2) \)?
• We computed the derivative (in terms of $x$ and $y$) at $x = 2$ without actually having an expression for $y$ in terms of $x$.

• Pretty Cool!

• It’s crucial to **differentiate first and evaluate second**.

• If we were to evaluate at $x = 2$ and $y = 1$ in

\[ y^3 + 7y = x^3 \]

we’d get

\[ 8 = 8 \]

• Differentiating in that equation gives

\[ 0 = 0 \]

which is true but does not tell us about $y'(2)$. 
• Profound application:

\[ \frac{d}{dx} x^r = r x^{r-1} \]

for all rational numbers \( r \).

• At this stage, technically we know the power rule only if \( r \) is an integer.
• Let’s do another example where we can get the same answer both ways and practice differentiation as well.

• Example 1, simplified:

\[ x^2 y - y = x^3 - 1 \]
Another Example. Problem 47, page 135, textbook. The graph of

\[ x^2 - xy + y^2 = 16 \]

is a tilted ellipse as shown in Figure 1. Find the equations of the tangents at the \( x \) intercepts.