Math 1210-23

Notes of 2/9/24

- We are heading into some tricky territory and will spend two meetings on section 2.7 of our textbook.
- Recall our differentiation rules:

$(kf)'$	$=$	kf'	**Constant Multiple Rule**
$(f+g)'$	$=$	$f'+g'$	**Sum Rule**
$(x^n)'$	$=$	nx^{n-1}	**Power Rule (n integer)**
$(fg)'$	$=$	$f'g+fg'$	**Product Rule**
$\left(\frac{f}{g}\right)'$	$=$	$\frac{f'g-fg'}{g^2}$	**Quotient Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\sin x$	$=$	$\cos x$	**Sine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\cos x$	$=$	$-\sin x$	**Cosine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}f\big(g(x)\big)$	$=$	$f'\big(g(x)\big)g'(x)$	**Chain Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x}$	$=$	$\frac{1}{2\sqrt{x}}$	**Square Root Rule**

2.7 Implicit Differentiation

- As usual, this is only a small extension of what we did previously. But it may still surprise you.
- We've been talking about how to differentiate, using our differentiation rules.
- That's what we'll continue to do today.
- But we won't necessarily assume that our function is defined explicitly.
- Example:

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

- Example: Do

$$f(x) = \sqrt{1 - x^2}$$

explicitly and implicitly

- Another example:

$$y = y(x), \qquad y^3 + 7y = x^3$$

- We think of y as a function of x
- It's easy to check that the point $(2, 1)$ is on the graph.
- What is $y'(2)$?

- We computed the derivative (in terms of x and y) at $x = 2$ without actually having an expression for y in terms of x.
- Pretty Cool!
- It's crucial to **differentiate first and evaluate second**.
- If we were to evaluate at $x = 2$ and $y = 1$ in

$$y^3 + 7y = x^3$$

  we'd get

$$8 = 8$$

- Differentiating in that equation gives

$$0 = 0$$

  which is true but does not tell us about $y'(2)$.

- Profound application:

$$\frac{\mathrm{d}}{dx}x^r = rx^{r-1}$$

  for all rational numbers r.

- At this stage, technically we know the power rule only if r is an integer.

- Let's do another example where we can get the same answer both ways and practice differentiation as well.
- Example 1, simplified:

$$x^2y - y = x^3 - 1$$

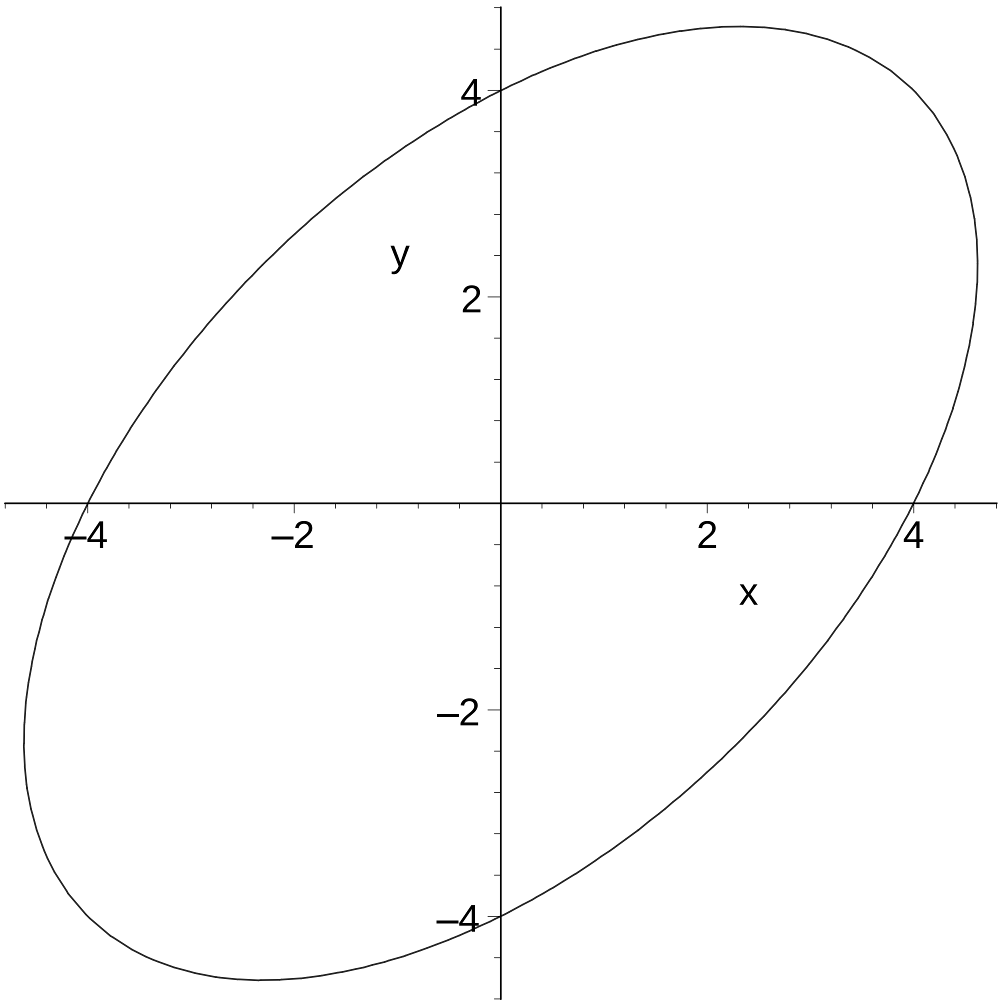

Figure 1. A tilted ellipse.

- Another Example. Problem 47, page 135, textbook. The graph of

$$x^2 - xy + y^2 = 16$$

is a tilted ellipse as shown in Figure 1. Find the equations of the tangents at the x intercepts.