Math 1210-23

Notes of 2/7/24

2.6 Higher Order Derivatives

- Recall our differentiation rules:

$(kf)' = kf'$		**Constant Multiple Rule**
$(f+g)' = f' + g'$		**Sum Rule**
$(x^n)' = nx^{n-1}$		**Power Rule (n integer)**
$(fg)' = f'g + fg'$		**Product Rule**
$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$		**Quotient Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$		**Sine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$		**Cosine Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$		**Chain Rule**
$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$		**Square Root Rule**

- Today's small extension of what we did previously: Of course we can differentiate more than once!
- Example:

$$f(x) = 3x^4 - 2x^3 + 5x^2 - 4x + 8$$

$$f(x) = x^{17}$$

- Compute the 17-th, 18-th, and n-th derivatives

- Differentiating a polynomial reduces the degree by 1.
- The $(n+1)$-th derivative of a polynomial of degree n, and all higher derivatives, are zero.
- Polynomials of degree n are all functions whose $(n+1)$-th derivative is zero.

Notation

- See the (incomplete) Table on page 126.
- Suppose
$$y = f(x).$$
Then some notations for various derivatives are:

1^{st}	y'	f'	$f'(x)$	$D_x f$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
2^{nd}	y''	f''	$f''(x)$	$D_x^2 f$	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$
3^{rd}	y'''	f'''	$f'''(x)$	$D_x^3 f$	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$
n-th	$y^{(n)}$	$f^{(n)}$	$f^{(n)}(x)$	$D_x^n f$	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$

- Another example:

$$f(x) = \sin x$$

- The fourth derivative of sine (or cosine) equals the function itself:

$$y^{(4)} = y$$

- This is our first example of a **Differential Equation**.

- Another Example:

$$y = \sin 2x$$

- Recall the height formula from preCalculus:

$$h(t) = -16t^2 + v_0 t + h_0$$

  where v_0 is the initial velocity and h_0 is the initial height.

- The velocity is the derivative of height and acceleration is the derivative of velocity.

- The third derivative of location (height) is sometimes called **jerk**.

- The jerk you feel when coming to a stop in a car is a jump discontinuity in the second derivative.

- Richard Nixon: "Inflation is still increasing. But the rate at which it is increasing is decreasing".

- What derivative (of cost) is he talking about?

- Differentiating a polynomial simplifies it.
- Differentiating a rational function makes it more complicated.
- Example, compute the first few derivatives of

$$f(x) = \frac{1}{x^2 + 1}$$

- Sometimes we can simplify differentiation with suitable tricks.
- Example: Compute the fourth derivative of

$$f(x) = \frac{x^4 + x^3 + x^2 + x + 1}{x}$$

- What about

$$f(x) = \frac{x^4 + x^3 + x^2 + x + 1}{x - 1}$$

(exercise!)

- Differentiation also tends to make trigonometric functions more complicated.
- Example:

$$f(x) = \tan x = \frac{\sin x}{\cos x}.$$