2.1-2 The Derivative

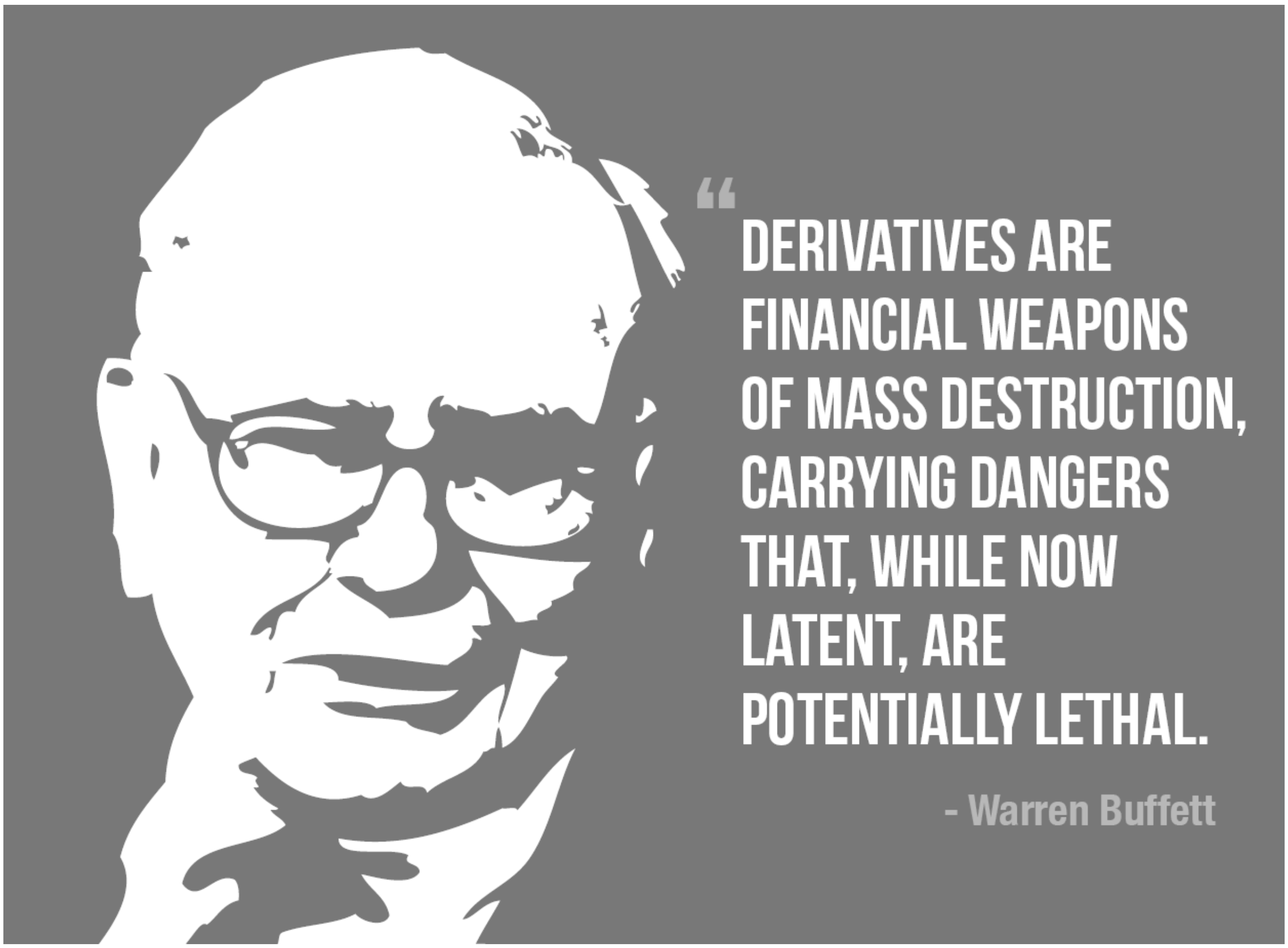

Figure 1. Warren Buffett on Derivatives.

- We need a fresh start.
- Remember

  Concept $\longrightarrow$ Definition $\longrightarrow$ Properties $\longrightarrow$ Work
- Recall our discussion of velocity:
- $s(t) =$ location at time t
- $v(t) =$ velocity at time t

$$v(t) \approx \frac{s(t+h) - s(t)}{h}$$

- We consider what happens as h approaches zero.
- Well, now that we know about limits, we can define:

$$v(t) = \lim_{h \longrightarrow 0} \frac{s(t+h) - s(t)}{h}$$

- we say that velocity is the **derivative** of location.
- In general, given a function f, its derivative f' is another function defined by

$$f'(t) = \lim_{h \longrightarrow 0} \frac{f(t+h) - f(t)}{h}$$

- This is the heart of Calculus. It's crucial to understand this definition and concept both algebraically and geometrically.

The Geometry

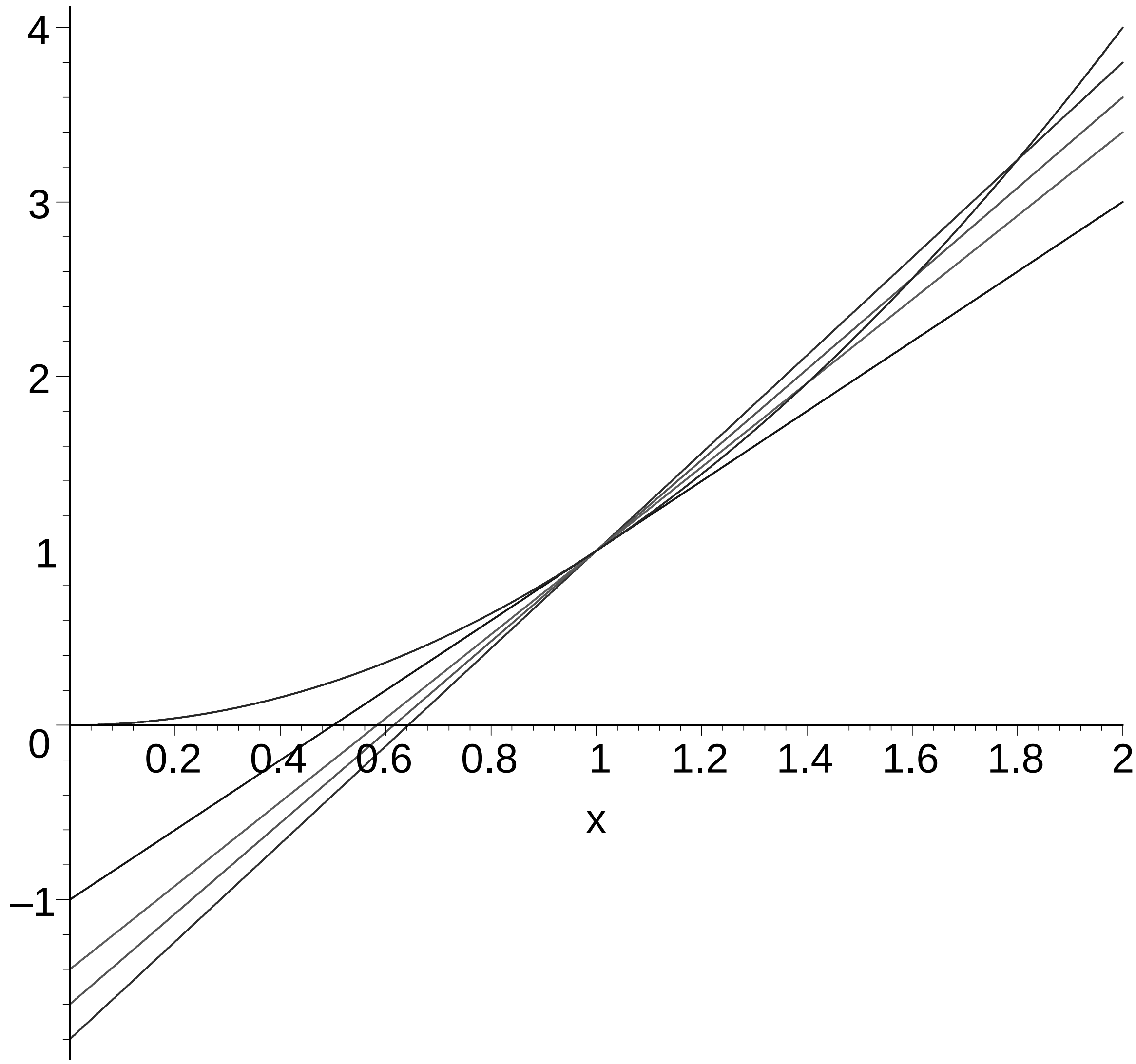

Figure 2. Secants and Tangent.

- The derivative gives the slope of the tangent (line) as the limit of the slopes of the secants (lines).
- The derivative is a function!

- For the rest of today we'll do some examples.
- $f(x) = x^2$ (Review)

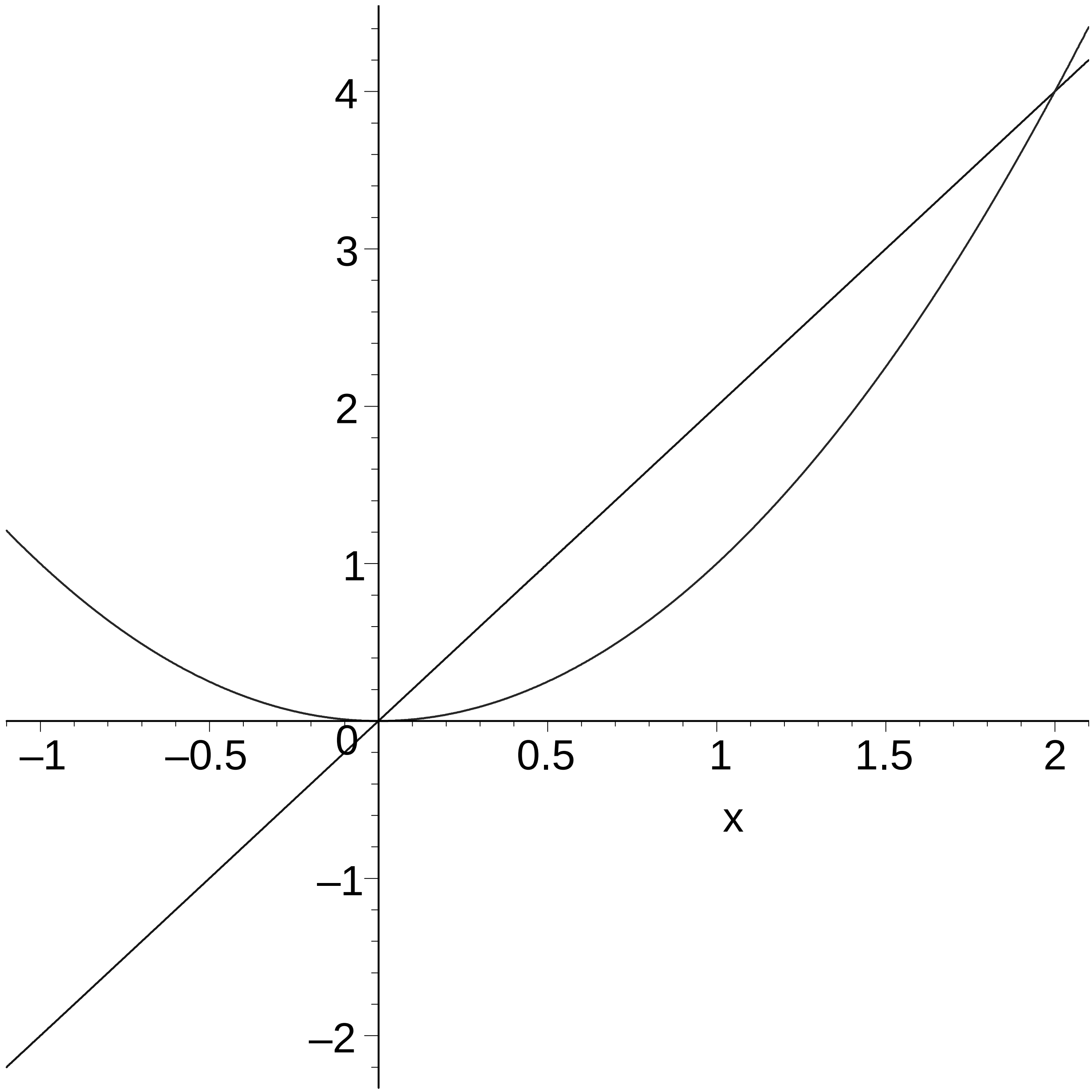

Figure 3. $f(x) = x^2$ and $f'(x) = 2x$.

- General principle: apply a new concept to a familiar context.
- The derivative of a constant should be zero:

- The derivative of a linear function should be constant.

- $f(x) = x^3$

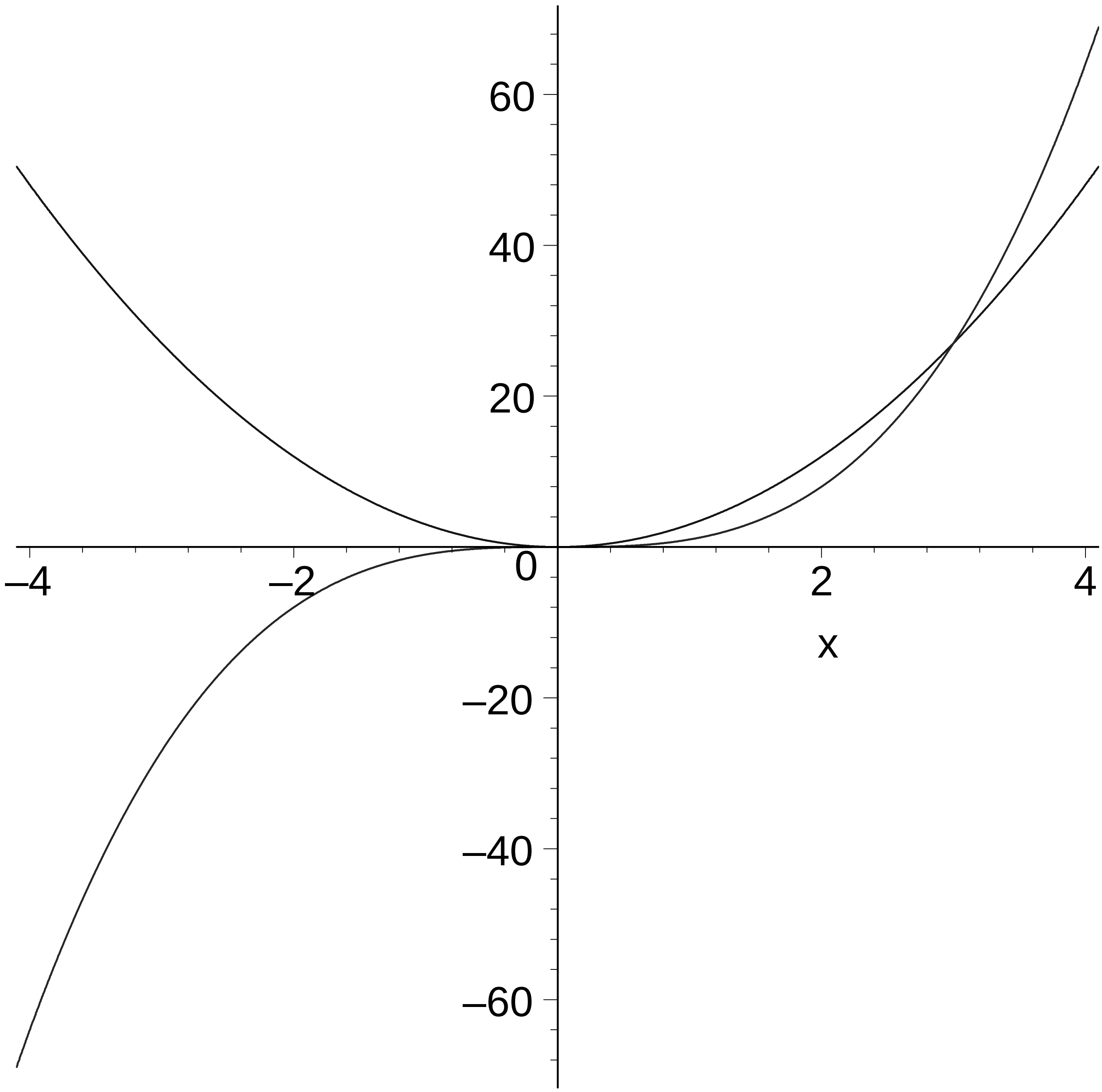

Figure 4. $f(x) = x^3$ and $f'(x) = 3x^2$.

- Note the discrepancy in the horizontal and vertical scales.

- $f(x)=\sqrt{x}$

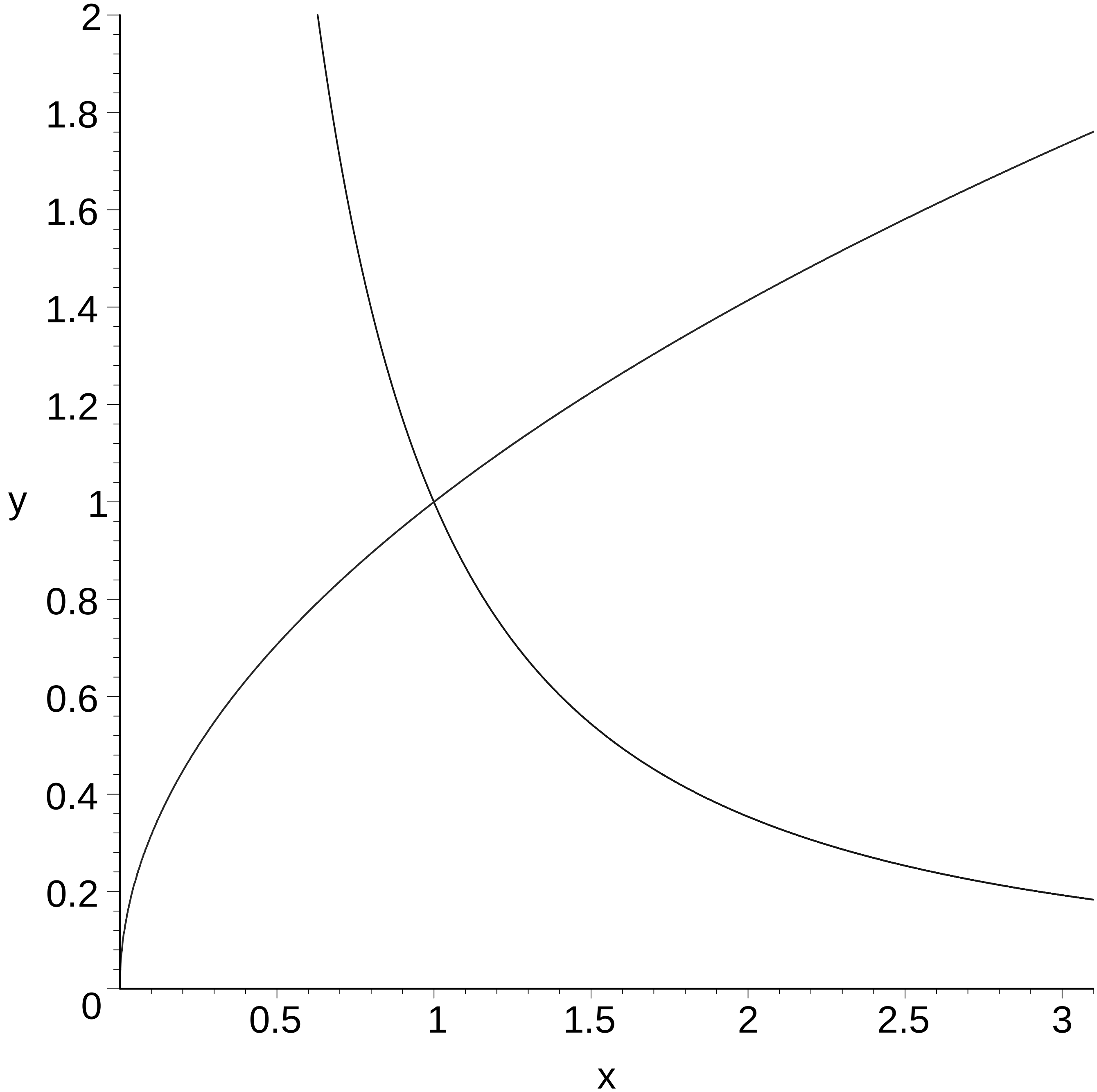

Figure 5. $f(x) = \sqrt{x}$ and $f'(x) = \frac{1}{2\sqrt{x}}$.

- $f(x) = \frac{1}{x}$

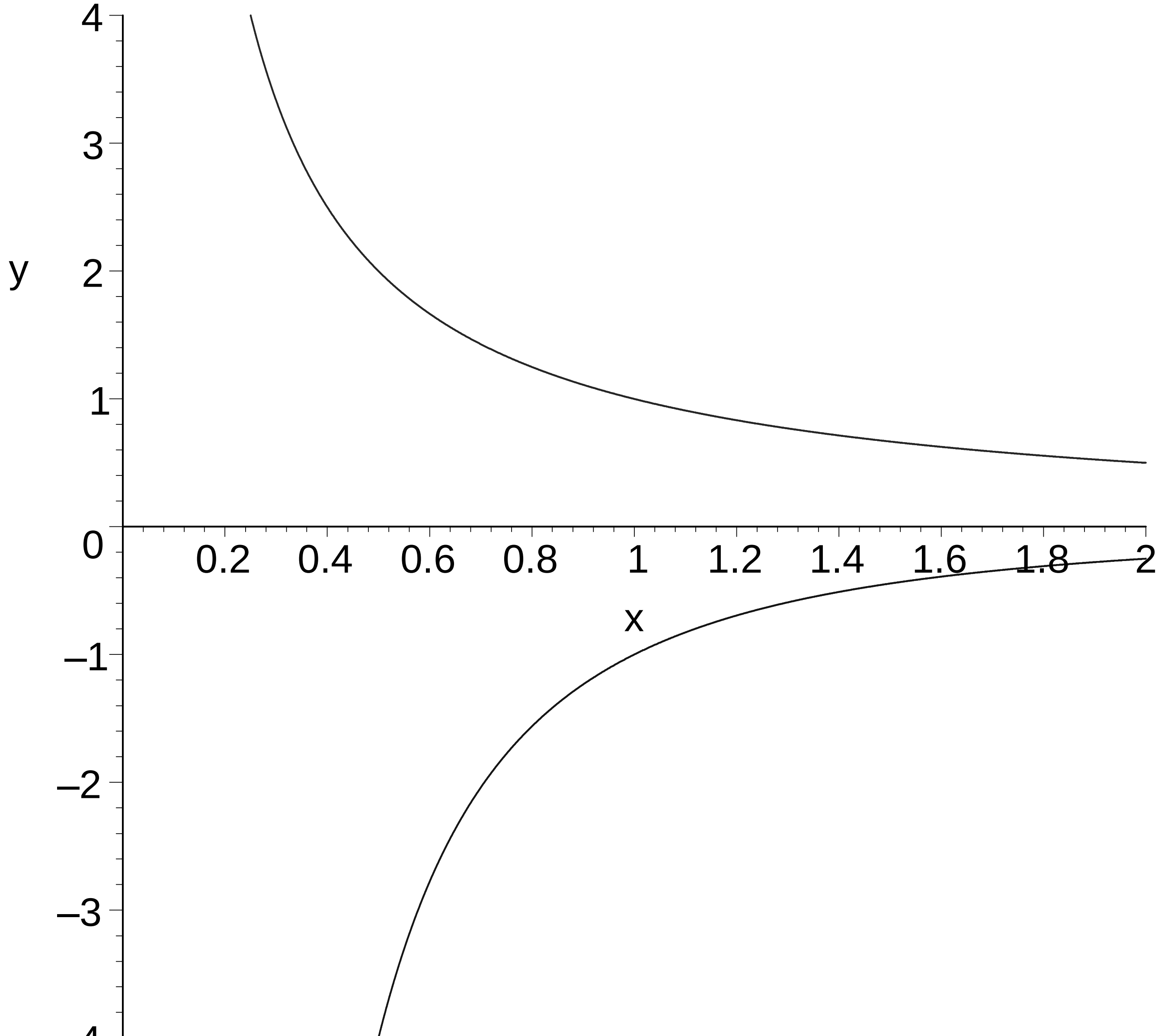

Figure 6. $f(x) = \frac{1}{x}$ and $f'(x) = -\frac{1}{x^2}$.

- $f(x) = 3x^2$

- $f(x) = x^2 + x^3$