Corrections to "Intro. to Homological Algebra" by C. Weibel

Cambridge University Press, paperback version, 1995

p.2 line -12: d_{n-1} should be d_n

p.4 lines 5,6: $V - E - 1$ should be $E - V + 1$ (twice)

p.4 lines 7,8: all 5 occurrences of v_0 should be replaced by v_1.

p.6, line 7 of Def. 1.2.1: "non-abelian" should be "non-additive"

p.12 line 1: $B \to C$ should be $B \xrightarrow{p} C$

p.12 line 9: "so is coker$(f) \to$ coker(g)" should be "so is coker$(g) \to$ coker(h)"

p.13 line -1: $Z_{n-1}(b)$ should be $Z_{n-1}(B)$

p.15 line 11: $B(-1)$ should be $B[-1]$

p.18 line 3: Replace the sentence "Give an example..." with: "Conversely, if C and $H_*(C)$ are chain homotopy equivalent, show that C is split."

p.18 line 18: Replace $i = 1, 2$ with $i = 0, 1$

p.21 Ex.1.5.3: Add extra paragraph: If $f : B \to C$, $g : C \to D$ and $e : B \to C$ are chain maps, show that e and gf are chain homotopic if and only if there is a chain map $\gamma = (e, s, g)$ from cyl(f) to D. Note that e and g factor through γ.

p.26 line -10: { let} $C^\infty(U)$ be ... that C^∞ is a sheaf...

p.27 line 7: the contour integral should be $\frac{1}{2\pi i} \oint f'(z)dz/f(z)$, not $\frac{1}{2\pi i} \oint f(z)dz$.

p.29 line 17: should read "[Freyd, p. 106], every small full abelian subcategory of $\mathcal{L}$ is equivalent to a full abelian subcategory of the category R-**mod** of modules over the ring"

p.32 line 1: 2.6.3 should be 2.6.4

p.34 line -14 (Ex. 2.2.1): Add this sentence before the hint: "Their brutal truncations $\sigma_{\geq 0}P$ form the projective objects in $\mathbf{Ch}_{\geq 0}$."

p.35 line 8: replace "chain map" by "quasi-isomorphism"

p.40 line -8: the map F should be f

p.47 line -6: the m^{th} *syzygy*

p.49 line 1: $L_n(f)$ should be $L_nF(f)$

p.49 line -11 (Ex. 2.4.4): Replace "the mapping cone cone(A) of exercise 1.5.1" by the following text: "$\sigma_{\geq 0}$cone$(A)[1]$, where cone(A) is the mapping cone of exercise 1.5.1. If $\mathcal{A}$ has enough projectives, you may also use the projective objects in $\mathbf{Ch}_{\geq 0}(\mathcal{A})$, which are described in Ex. 2.2.1."

p.56 line 13: (1) and (2) should be switched

p.57 lines 2,-10: a is the image ('a' should be 'a' twice)

p.57 line 4: $a_{jk} \in A_j$ should be $a_j \in A_j$

p.66 line 9: $pB = 0$ should be $pb = 0$.

p.74 Exercise 3.3.1: $\cdots \cong \mathbb{Z}_{p^\infty}$ should be $\cdots \cong (\mathbb{Q}/\mathbb{Z}[1/p]) \times \hat{\mathbb{Q}}_p/\mathbb{Q}$.

p.74 Exercise 3.3.5: In the display, replace A/pA with A^*/pA^* and delete the final '$= 0$'. On the next line (line -1), 'A is divisible' should be 'A^* is divisible, i.e., A is torsionfree'.

p.82 line 6: the above proof can be modified to show ...

p.82 line 8: However, the proof of Lemma 3.5.3 breaks down for other abelian categories, because one needs to assume that $\mathcal{A}$ has enough projectives (or has a generating family of projectives) in order to use element-theoretic methods with infinite products (see [EM, 2.7]). I do not know if Lemma 3.5.3 or Proposition 3.5.7 holds without an assumption like this.

p.82 line -8: 'complete' should be 'complete and Hausdorff'

p.85 line -4: Then Tot$(C) =$ Tot$^\Pi(C)$ is ...

p.86 line 2: "nonzero columns." (not rows) p.89 line 8: $\}\otimes\{$ should be $\}\oplus\{$.

p.97 line 14: Add sentence:

"If in addition R is finite-dimensional over a field then R is quasi-Frobenius $\Leftrightarrow$ R is Frobenius."

p.101 line -5: $n<\infty$ should be $d<\infty$

p.102 lines 3,4: $\le 1+n$ should be $\le 1+d$ twice

p.122 line 9: $-(r+1)/r$ should be $-(r-1)/r$

p.124 line 7: E^∞_{0n} is a quotient of E^a_{0n} and each E^∞_{n0} is a subobject of E^a_{n0}.

p.124 line -7: $0\to E^2_{0n}\to H_n\to E^2_{1,n-1}\to 0$.

p.127 line 14 (**): $(-1)^{p_1}$ should be $(-1)^{p_1+q_1}$

p.132 line -3: $F_sH_n(C)$ should be F_sC_n

p.134 line 8: The filtration on the complex C' is bounded below, the one on C'' ...

p.135 line 12: the superscripts r should be $r+1$, viz.,

$$\cdots E^{r+1}_{p+r}(\mathrm{cone}\, f)\to E^{r+1}_p(B)\to E^{r+1}_p(C)\to E^{r+1}_p(\mathrm{cone}\, f)\to E^{r+1}_{p-r}(B)\cdots.$$

p.135 line 18: Insert after $d(c)=0$: "(This assumes that (AB5) holds in $\mathcal{A}$.)"

p.135 line -13: after 'exhaustive' add: "and that $\mathcal{A}$ satisfies (AB5)."

p.135 line -4: replace text starting with E^1_{p0} to read: E^1_{p0} is $\bar{C}_p=C_p/(F_{p-1}C_p+d(F_pC_{p+1})$; $\bar{C}$ is the top quotient chain complex of C, and $d^1_{p0}:E^1_{p0}\to E^1_{p-1,0}$ is induced from $d:C_p\to C_{p-1}$.

p.136 line -9,-8: let $F_{-p}C$ be 2^pC $(p\ge 0)$.

p.136 line -7: "Each row" should be "Each column"

p.137 Cor. 5.5.6 should read: If the spectral sequence weakly converges, then the filtrations on $H_*(C)$ and $H_*(\widehat{C})$ have the same completions.

p.143 line 15: $H_q(A)$ should be $H_q(Q)$

p.152 line 8: $\xrightarrow{\otimes_S R}$ should read $\xrightarrow{\otimes_R S}$.

p.154 line -2: $\mathcal{E}=\mathcal{E}^{a+1}$ and $\mathcal{E}^r$ denotes the $(r-a-1)^{st}$ derived couple

p.154 line -1: $j^{(r)}$ has bidegree $(1-r,r-1)$

p.155 line 2: remove $\xrightarrow{i} D$ from diagram to read: $E^r_{pq}\xrightarrow{k}D^r_{p-1,q}\xrightarrow{j^{(r)}}E^r_{p-r,q+r-1}$.

p.155 line 6: starting with E^{a+1}.

p.168 line -1: $H_{1-n}(G;A)$ should be $H_{-1-n}(G;A)$

p.177 line 13: If m is odd, every automorphism of D_m stabilizing C_m is inner.

p.185 fourth line of proof of Classification Theorem: $\beta(1)$ should be $\beta(1)=1$

p.186 line -3: b_hg should be b_hh.

p.191 Cor. 6.7.9: ... of G on H induces an action of G/H on $H_*(H;\mathbb{Z})$ and $H^*(H;\mathbb{Z})$.

p.191 line -3: complex of (space missing)

p.193 line 19: delete '$\beta\sigma=0$' so it reads '$(\sigma^2=0)$'

p.193 lines -3, -8; and p.194 line 7: "cocommutative" should be "coassociative"

p.196 line -4: If H is in the center of G and A is a trivial G-module then G/H acts trivially ...

p.201 Exercise 6.9.2: If ... and ... are central extensions, and X is perfect, show ...

p.203 lines 1–2: When $\mathbb{F}_q$ is a finite field, and $(n,q)\ne(2,2),(2,3),(2,4),(2,9),(3,2),(3,4),(4,2)$, we know that $H_2(SL_n(\mathbb{F}_q);\mathbb{Z})=0$ [Suz, 2.9]. With these exceptions, it follows that

p.213 Exercise 6.11.11: ... Show that for $i\ne 0$:

$$H^i(G;\mathbb{Z})=\begin{cases}\mathbb{Z}_{p^\infty} & i=2\\ 0 & \text{else.}\end{cases}$$

p.226: Line 3 of Exercise 7.3.5 should read: δ-functors (assuming that that k is a field, or that N is a projective k-module):

p.238 lines 4–7: Replace these two sentences (Show that...it suffices to show that ...= 0.) by:

Conversely, suppose that $\mathfrak{g} = \mathfrak{f}/\mathfrak{r}$ for some free Lie algebra $\mathfrak{f}$ with $\mathfrak{r} \subseteq [\mathfrak{f}, \mathfrak{f}]$, and $\mathfrak{g}$ is free as a k-module. Show that if $H^2(\mathfrak{g}, M) = 0$ for all $\mathfrak{g}$-modules M then $\mathfrak{g}$ is a free Lie algebra. *Hint:* It suffices to show that ...= 0.

p.256 line 8: identity (not identify)

p.258 lines 1, 20 and -7: 'combinational' should be 'combinatorial'

p.262 line 18: $g_r = g(\sigma_r u)^{-1}$

p.265 add to end of Exercise 8.3.3:
Extend exercise 8.2.5 to show that a homomorphism of simplicial groups $G \to G''$ is a Kan fibration if and only if the induced maps $N_n G \to N_n G''$ are onto for all $n > 0$. In this case there is also a long exact sequence, ending in $\pi_0(G'')$.

p.266 line 14: that $\sigma_i(x_i) \neq 0$, then $y = y - \sigma_i \partial_i y = \sum_{j>i} \sigma_j(x'_j)$. By induction, $y = 0$. Hence $D_n \cap N_n = 0$.

p.267 line 5: fix subscript on sum: $d\sigma_p(x) = \sum_{p+2}^{n}$

p.267 line 6: $d\sigma_p^2(x) + \sigma_p d\sigma_p(x) = \sum_{i=p+2}^{n+1} \cdots + \sum_{i=p+2}^{n}$

p.267 line 7: $= (-1)^p \sigma_p(x)$.

p.267 line 8: Hence $\{s_n = (-1)^p \sigma^p\}$

p.278 display on line 8: 1 should be subtracted from the subscripts: $\sigma^h_{\mu(n)-1} \cdots \sigma^h_{\mu(p+1)-1} \sigma^v_{\mu(p)-1} \cdots \sigma^v_{\mu(1)-1}$

p.280 lines 10–11: η: $1_{\mathcal{C}} \to UF$ and ... ε: $FU \to 1_{\mathcal{B}}$. (switch $\mathcal{B}$ and $\mathcal{C}$)

p.287 lines -5, -4: "is an exact sequence" should be "is a sequence" and "is also exact" should be "is exact"

p.291 line -2: ... to $(R/I)^d$. If each $x_i R \subset R$ is k-split then:

p.294 line -8: If M is an R-module, ('a k' should be 'an R')

p.295 line -4 (display): $D^1(R, M)$ should be $D^1(R/k, M)$

p.296 8.8.6: "If k is a field" should be "If R is a field"

p.297, line -9: the sequence should read

$$\cdots \to D_{n+1}(R/K, M) \to D_n(K/k, M) \to D_n(R/k, M) \to D_n(R/K, M) \to D_{n-1}(K/k, M) \to \cdots.$$

p.298 line 2 of 8.8.7: *Commalg* should be in roman font: **Commalg**

p.301 lines 4–5: the ranges should be "if $0 < i \leq n$" and "if $i = n+1$" respectively.

p.307 line -1 should read:

As $\mathrm{Tor}_1^{R^e/k}(R^e, M) = 0$, the long exact relative Tor sequence (Lemma 8.7.8) yields

p.322: On line 1, insert "If $1/2 \in k$," before "$\Omega^*_{R/k}$ is the free graded-commutative" and (on line 4) add the sentence: In general, $\Omega^*_{R/k}$ is the free alternating R-algebra generated by $\Omega^1_{R/k}$.

p.354 line -6: Add sentence: It also follows from the Connes-Karoubi theorem on noncommutative de Rham homology in C.R. Acad. Sci. Paris, t. 297 (1983), p. 381–384.

p.359 Exercise 9.9.5: This is wrong; replace it with:
Exercise 9.9.5 (Grauert-Kerner) Consider the artinian algebra $R = k[x, y]/(\partial f/\partial x, \partial f/\partial y, x^5)$, where $f = x^4 + x^2y^3 + y^5$. Show that $I = (x, y)R$ is nilpotent, and f is a nonzero element of $H^0_{dR}(R)$ which vanishes in $H^0_{dR}(R/I)$.

p.370 line -6: [ho-]"mopy" should be [ho-]"motopy" and $b,''$ should be b'',

p.376 line -10: diagram ... commutes up to chain homotopy.

p.381 line 6: '(left)' should be '(right)'

p.382 lines-6,-7 (10.3.8): the two occurrences of 'right' should be 'left'

p.384 lines 9, 11: 'left' fraction should be 'right' fraction p.384 line 13: Replacing 'right' by 'left'

p.385 line 1: $\mathcal{B}$ is a small category and $\text{Ext}(A, B)$ is a set for all $A \in \mathcal{A}$, $B \in \mathcal{B}$. Then show that...

p.386 lines 7–9: six occurrences of g should be v: ...there should be a $v : X \to Z$... $f - g = uv$. Embed v in an exact triangle (t, v, w) ... Since $vt = 0$, $(f - g)t = uvt = 0$, ...

p.386 lines 14–18: Replace the two sentences "Given us_1^{-1} : ... triangle in **K**" with: "The exact triangles in $S^{-1}\mathbf{K}$ are defined to be those triangles which are isomorphic, in the sense of (TR1), to the image under $\mathbf{K} \to S^{-1}\mathbf{K}$ of an exact triangle in **K**."

p.386 line 20: replace "straightforward but lengthy; one uses the fact" with "straightforward; one uses (TR3) and the fact that..."

p.387 line -12 (10.4.5): delete 'well-powered' (Gabber points out that this condition is superfluous).

p.405 line -2: a natural homomorphism in $\mathbf{D}(R)$, which is an isomorphism if either each C_i is fin. gen. projective or else A is quasi-isomorphic to a bounded below chain complex of fin. gen. projective R-modules:

p.420 line 13: in exercise 6.11.3 (not 6.11.4)

p.427 line -1: $I \in I$ should be $i \in I$

p.428 line 16: $F_i \to F_i \to C$ should be $F_j \to F_i \to C$

p.431 line 6: 'functions' should be 'functors.'

p.439 add entry to Index under "double chain complex":
Connes' — $\mathcal{B}$. *See* Connes' double complex.

p.444, under 'Lie group': [page] 158 should be 159

p.445, line -6: 'Øre' should be 'Ore'

p.448 column 2: lines 29-30 should only be singly indented ("— of" refers to spectral sequence)

References

[EM] Eilenberg, S., and Moore, J. "Limits and Spectral Sequences." *Topology* **1** (1961): 1–23.