Facts for Final Exam

Math 2270, Fall 2006

Prove the following facts:

Chapter §8.

(1) (8.1.2) Consider a symmetric matrix A. If $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of A with distinct eigenvalues λ_1 and λ_2, then $\vec{v}_1$ḑ$ot\vec{v}_2 = 0$.

(2) (8.1.3) A symmetric $n \times n$ matrix A has n real eigenvalues if they are counted with their algebraic multiplicities.

(3) (8.2.2) Consider a quadric form $q(\vec{x}) = \vec{x} \cdot A\vec{x}$, where A is a symmetric $n \times n$ matrix. Let $\mathcal{B}$ be an orthonormal eigenbasis for A, with associated eigenvalues $\lambda_1, \ldots, \lambda_n$. Then $q(\vec{x}) = \lambda_1 c_1^2 + \lambda_2 c_2^2 + \cdots + \lambda_n c_n^2$, where c_i are the coordinates of $\vec{x}$ with respect to $\mathcal{B}$.

(4) (8.3.3) Let $L(\vec{x}) = A\vec{x}$ be a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$. Then there exists an orthonormal basis $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ of $\mathbb{R}^m$ such that

(a) Vectors $L(\vec{v}_1), L(\vec{v}_2), \ldots, L(\vec{v}_m)$ are orthogonal, and

(b) The lengths of vectors $L(\vec{v}_1), L(\vec{v}_2), \ldots, L(\vec{v}_m)$ are the singular values $\sigma_1, \sigma_2, \ldots \sigma_m$ of matrix A.