Exam # 3 **Fall 2006** **MATH 2270-03**

Instructor: Oana Veliche
Time: 50 minutes

NAME:___________________________________

ID#:_____________________________________

INSTRUCTIONS
(1) Fill in your name and your student ID number.
(2) Justify all you answers. Correct answers with no justification will not be given any credit.
(3) No books, notes or calculators may be used.

Problem	1	2	3	4	5	6	Total
Max. points	15	30	10	15	10	20	100
Points							

1. Problem. Let A be a 3×3 matrix with rows $\vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\det(A) = 2$. Find the following determinants:

(5 points)(a) $\det \begin{bmatrix} -\vec{v}_2 \\ 2\vec{v}_1 \\ \vec{v}_1 + \vec{v}_3 \end{bmatrix} =$

(5 points) (b) $\det(AA^T) =$

(5 points) (c) $\det(3A) =$

2. Problem. Consider the matrix $U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$. This matrix has the eigenvalues:

$$\lambda_1 = 0, \quad \lambda_2 = \lambda_3 = \lambda_4 = 2.$$

(5 points) (a) Give a short reason for why is 0 an eigenvalue.

(5 points) (b) Find the eigenspace E_2.

(5 points) (c) Is the matrix U diagonalizable? (Hint: You might not need to find all eigenspaces.)

Using that the linear transformation $T\colon \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ given by $T(M) = M + M^T$ has the $\mathcal{U}$-matrix U above, where $\mathcal{U} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ find the following:

(5 points) (d) $\det(T)$.

(5 points) (e) The eigenvalues of T.

(5 points) (f) An eigenspace of the transformation T for one eigenvalue, of your choice, found in (e).

3. Problem. Consider the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}$. Find the following:

(5 points) (a) The complex eigenvalues of the matrix A.

(5 points) (b) Find an invertible matrix S and a matrix $B = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that $A = SBS^{-1}$.

4. Define the following notions:

(5 points) (a) Rotation matrix.

(5 points) (b) Geometric multiplicity of an eigenvalue.

(5 points) (b') Knowing that $\lambda = 1$ is an eigenvalue for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$, find the geometric multiplicity of $\lambda = 1$.

(10 points) **5. Prove the following fact.** Let A and B be two $n \times n$ matrices. If B is obtained from A by a row swap, then $\det(B) = -\det(A)$.

6. True or false ? Justify all your answers.

(5 points) (a) There exists an invertible matrix A with $\operatorname{adj}(A)$ not invertible.

(5 points) (b) There exists a square matrix A with all eigenvalues negative real numbers and $\mathrm{tr}(A) = 2$.

(5 points) (c) Consider the vectors: $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. The 2-volume of the 2-parallelepiped defined by these two vectors is 2.

(5 points) (d) If λ is an eigenvalue of an $n \times n$ matrix A, then λ is an eigenvalue of the matrix A^T.