

Section 7.3

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(a) $f(0.5, 4) = \sqrt{0+5+4} = 3$

(b) $f(6, 8, -3) = \sqrt{6+8-3} = \sqrt{11}$

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(a) $F(0.09, 60) = 500 \left(1 + \frac{0.09}{12}\right)^{60} \approx 782.84$

(b) $F(0.14, 240) = 500 \left(1 + \frac{0.14}{12}\right)^{240} \approx 8090.135$

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(a) $A(500, 0.10, 5) = 500 \cdot e^{0.10 \cdot 5} = 500 e^{0.5} \approx 824.36$

(b) $A(1500, 0.12, 20) = 1500 e^{0.12 \cdot 20} = 1500 e^{2.4} \approx 16534.76$

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$g(x, y) = \int_x^y \frac{1}{t} dt$

(a) $g(4, 1) = \int_4^1 \frac{1}{t} dt = \ln|t| \Big|_4^1 = \ln 1 - \ln 4 = -\ln 4$

(b) $g(6, 3) = \int_6^3 \frac{1}{t} dt = \ln 3 - \ln 6 = \ln \frac{3}{6} = -\ln 2$

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(a) $f(x+\Delta x, y) = 3(x+\Delta x) \cdot y + y^2$

$= 3xy + 3(\Delta x)y + y^2$

(b) $\frac{f(x, y+\Delta y) - f(x, y)}{\Delta y}$

$= \frac{[3x(y+\Delta y) + (y+\Delta y)^2] - [3xy + y^2]}{\Delta y}$

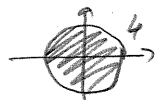
$= \frac{3x\Delta y + \Delta y(2y + \Delta y)}{\Delta y}$

$= \frac{\Delta y(3x + 2y + \Delta y)}{\Delta y}$

$= 3x + 2y + \Delta y$

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$f(x, y) = \sqrt{16 - x^2 - y^2}$



$16 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 16$

Domain = all points inside and on the circle $x^2 + y^2 = 16$

Range $[0, 4]$

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$f(x, y) = x^2 + y^2 - 1$

Domain: \mathbb{R}^2

Range $[-1, \infty)$

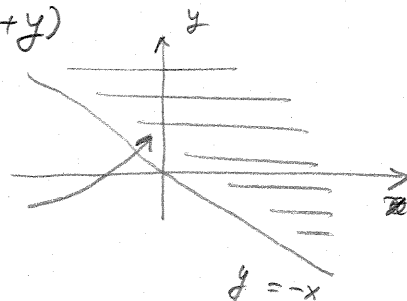
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$f(x, y) = \ln(x+y)$

$x+y > 0$

Domain: semiplane

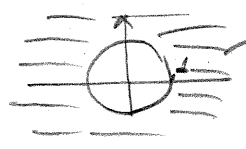
Range \mathbb{R}



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f(x,y) = sqrt(x^2+y^2-1)

x^2+y^2-1 >= 0 => x^2+y^2 >= 1



Domain = outside and on the circle x^2+y^2=1

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f(x,y) = x^2 + y^2/4 -> (e) because

x^2 + y^2/4 = c is an ellipse.

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f(x,y) = e^(1-x^2-y^2)

1-x^2-y^2 = c > 0

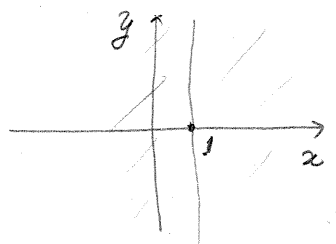
1-x^2-y^2 = ln c

x^2+y^2 = 1 - ln c 0 < c < e circle => (a)

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f(x,y) = 4y/(x-1)

x-1 != 0 x != 1

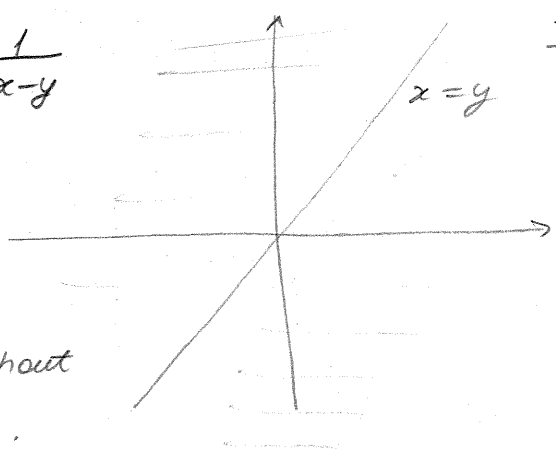


Domain = the plane without the line x=1

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g(x,y) = 1/(x-y)

x != y

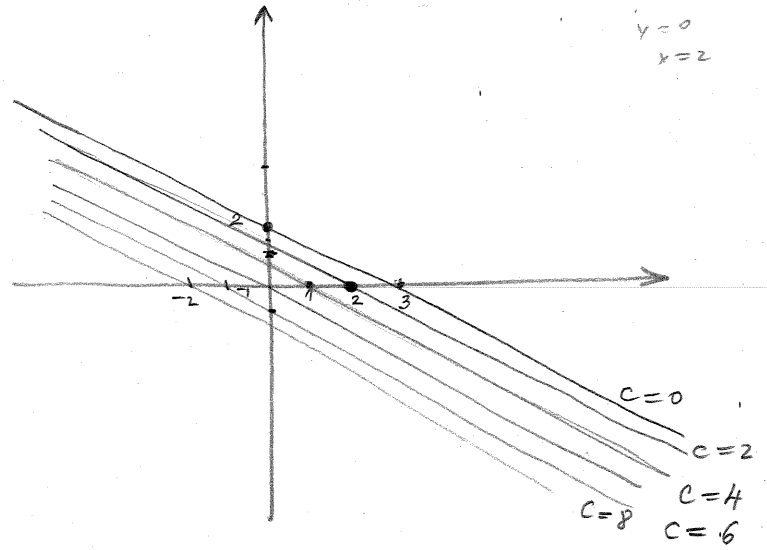


Domain = The plane without the line x=y.

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z = 6 - 2x - 3y c = 0, 2, 4, 6, 8, 10

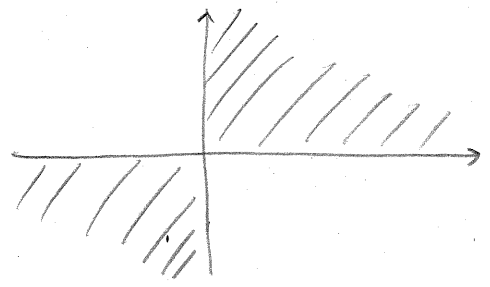
2x + 3y = 6 - c



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f(x,y) = sqrt(xy)

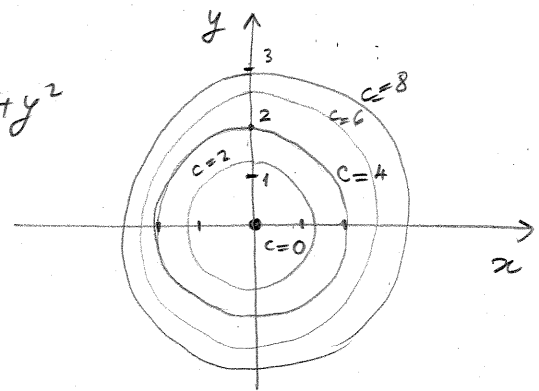
xy >= 0 => x >= 0 and y >= 0 OR x <= 0 and y <= 0



Domain: x and y axes together with the two shaded quarters of the plane.

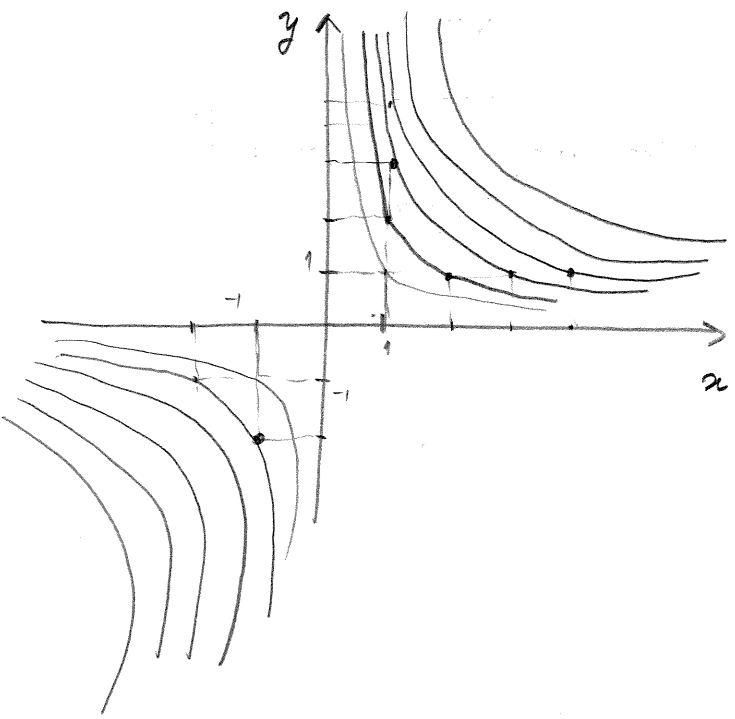
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f(x,y) = x^2 + y^2



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f(xy) = xy = c => y = c/x



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V(I, R) = 1000 [(1 + 0.10(1-R)) / (1+I)]^10

R \ I	0.00	0.03	0.05
0.00	\$2593.74	\$1929.99	\$1592.33
0.28	\$2004.23	\$1491.34	\$1230.42
0.35	\$1877.14	\$1396.77	\$1152.40

V(0, 0) = 1000 (1.1/1)^10 = \$2593.74

Section 7.4

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$$f(x,y) = x^2 - 3y^2 + 7$$

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = -6y$$

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$$\frac{\partial f}{\partial x} = -\frac{1}{2}x^{-3/2}, \quad \frac{\partial f}{\partial y} = 2 \cdot \frac{3}{2}y^{1/2} = 3y^{1/2}$$

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$$z = x\sqrt{y}$$

$$\frac{\partial z}{\partial x} = \sqrt{y}, \quad \frac{\partial z}{\partial y} = x \cdot \frac{1}{2\sqrt{y}}$$

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$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{y(x^2+y^2) - xy(2x)}{x^2+y^2}$$

$$= \frac{yx^2+y^3-2x^2y}{x^2+y^2}$$

$$= \frac{y(y^2-x^2)}{x^2+y^2}$$

By symmetry:

$$\frac{\partial f}{\partial y}(x,y) = \frac{x(x^2-y^2)}{x^2+y^2}$$

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$$z = xe^{x+y} = xe^x \cdot e^y$$

$$\frac{\partial z}{\partial x} = (e^x + xe^x) \cdot e^y = (1+x)e^{x+y}$$

$$\frac{dz}{dy} = xe^x \cdot e^y = xe^{x+y}$$

1

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$$g(x,y) = e^{x/y}$$

$$\frac{\partial g}{\partial x}(x,y) = e^{x/y} \cdot \frac{\partial}{\partial x} \left(\frac{x}{y} \right)$$

$$= e^{x/y} \cdot \frac{1}{y}$$

$$\frac{\partial g}{\partial y}(x,y) = e^{x/y} \frac{\partial}{\partial y} \left(\frac{x}{y} \right)$$

$$= e^{x/y} \cdot x(-1)(y^{-2}) = -\frac{xe^{x/y}}{y^2}$$

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$$g(x,y) = \ln \sqrt{x^2+y^2} = \frac{1}{2} \ln(x^2+y^2)$$

$$\frac{\partial g}{\partial x}(x,y) = \frac{1}{2} \cdot \frac{1}{x^2+y^2} \cdot 2x = \frac{x}{x^2+y^2}$$

By symmetry.

$$\frac{\partial g}{\partial y}(x,y) = \frac{y}{x^2+y^2}$$

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$$f_y(x,y) = \frac{\partial}{\partial y} (3x^2e^x \cdot ye^{-y})$$

$$= 3x^2e^x(e^{-y} - ye^{-y})$$

$$= 3x^2e^x(1-y) \cdot e^{-y}$$

$$= 3x^2(1-y)e^{x-y}$$

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$$g_y(x,y) = \frac{\partial}{\partial y} (3x \cdot e^{-x} \cdot y^2e^y)$$

$$= 3xe^{-x}(2ye^y + y^2e^y)$$

$$= 3xy(2+y)e^{y-x}$$

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f_x = 2x - 3y

f_x(1,-1) = 2 + 3 = 5

f_y = -3x + 2y

f_y(1,-1) = -3 - 2 = -5

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f_x = d/dx (e^{3xy}) = e^{3xy} * 3y

f_x(0,4) = e^0 * 3 * 4 = 12

f_y = e^{3xy} * 3x

f_y(0,4) = 0

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f_x = 1 / (x^2 + y^2) * 2x

f_x(1,0) = 2/1 = 2

f_y = 2y / (x^2 + y^2)

f_y(1,0) = 0

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w_x = 6xy - 5yz

w_y = 3x^2 - 5xz + 10z^2

w_z = -5xy + 20yz

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w_x = ye^{z^2}

w_x(2,1,0) = 1

w_y = xe^{z^2}

w_y(2,1,0) = 2

w_z = 2xye^{z^2}

w_z(2,1,0) = 0

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f_x = 9x^2 - 12y = 0 and f_y = -12x + 3y^2 = 0 implies 3x^2 = 4y and 4x = 3y^2

12x^3 = 12y^3 implies x^3 = y^3 implies x = y

3x^2 = 4x implies x = 0 or x = 4/3

(0,0) and (4/3, 4/3) are the solutions.

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f_x = 2x / (x^2 + y^2 + 1) = 0 implies x = 0

f_y = 2y / (x^2 + y^2 + 1) = 0 implies y = 0

(0,0) is the only solution

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dz/dx = 2x and dz/dx(-2,1) = -4 slope along x-direction

dz/dy = -2y and dz/dy(-2,1) = -2 slope along y-direction

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} (-6x^2y + 4y^3)$$

$$= -12xy$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y} (4x^3 - 6xy^2)$$

$$= -12xy$$

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$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} (6x - y) = 6$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} (-x) = 0$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} (-x) = -1$$

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$$f_{xx} = \frac{\partial}{\partial x} (4x^3 - 6xy^2) = 12x^2 - 6y^2$$

$$f_{xx}(1,0) = 12$$

$$f_{yy} = \frac{\partial}{\partial y} (-6x^2y + 2y) = -6x^2 + 2$$

$$f_{yy}(1,0) = -6$$

$$f_{xy} = f_{yx} = \frac{\partial}{\partial y} (4x^3 - 6xy^2) = -12xy$$

$$f_{xy}(1,0) = f_{yx}(1,0) = 0$$

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$$(a) \frac{\partial f}{\partial x} = 100 \cdot 0.6 x^{-0.4} y^{0.4}$$

$$\frac{\partial f}{\partial x}(1000, 500) = 60 \cdot \left(\frac{y}{x}\right)^{0.4}$$

$$= 60 \cdot \left(\frac{500}{1000}\right)^{0.4}$$

$$= 60 \left(\frac{1}{2}\right)^{0.4}$$

$$= 45.47 \text{ \$/unit}$$

$$\frac{\partial f}{\partial y} = 100 \cdot 0.4 x^{0.6} y^{-0.6}$$

$$= 40 \cdot \left(\frac{x}{y}\right)^{0.6}$$

$$\frac{\partial f}{\partial y} \Big|_{(1000, 500)} = 40 \cdot \left(\frac{1000}{500}\right)^{0.6}$$

$$= 40 \cdot 2^{0.6}$$

$$= 60.62 \text{ \$/unit}$$

