Exam # 3 **Fall 2005** **MATH 1100-05**

Instructor: Oana Veliche
Time: 1 hour

SOLUTIONS

NAME:______________________________

ID#:________________________________

INSTRUCTIONS
(1) Fill in your name and your student ID number.
(2) There are 10 problems, each worth 10 points.
(3) Justify all you answers. Correct answers with no justification will not be given any credit.
(4) Use (non-graphing) calculators only on the problems marked with $\boxed{c}$.
(5) A list of definitions and formulas is attached at the end.

Problem #	1	2	3	4	5	6	7	8	9	10	Total
# Points											

Problem 1. (a) Find the indefinite integral: $\int 3\sqrt{x} - \frac{1}{\sqrt{x}}\, dx.$ $= \ldots$

$$\ldots = \int 3x^{1/2} - x^{-1/2}\, dx$$

$$= 3 \cdot \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C$$

$$= 2x\sqrt{x} - 2\sqrt{x} + C$$

(b) Find the equation of the function f whose graph passes through the point $(1, 2)$, if

$$f' = 3\sqrt{x} - \frac{1}{\sqrt{x}}.$$

$$f(x) = \int 3\sqrt{x} - \frac{1}{\sqrt{x}}\, dx = 2x\sqrt{x} - \sqrt{x} + C$$

$$f(1) = 2 \cdot 1 \cdot 1 - 1 + C = 2$$

$$1 + C = 2 \implies C = 1$$

$$f(x) = 2x\sqrt{x} - 2\sqrt{x} + 1$$

☐c **Problem 2.** Find the change in profit if the marginal profit is given by

$$\frac{dP}{dx} = 0.4\left(1 - \frac{5000}{x}\right) = 0.4 - \frac{2000}{x}$$

and the number of units increases by 100 from $x = 5000$.

$$P(x) = \int_{5000}^{5100} 0.4 - \frac{2000}{x}\, dx$$

$$= 0.4x \Big|_{5000}^{5100} - 2000 \ln|x| \Big|_{5000}^{5100}$$

$$= 0.4\,(5100 - 5000) - 2000\left[\ln(5100) - \ln(5000)\right]$$

$$= 0.4 \cdot 100 - 2000 \ln\left(\frac{51}{50}\right)$$

$$= 40 - 2000 \cdot \ln\left(\frac{51}{50}\right)$$

$$= \$\, 0.39474540 7$$

Problem 3. Using the general power rule, or the method of substitution find the indefinite integral:

$$\int 3x(2x^2+1)^3\,dx. = \ldots$$

By the general power rule:

$$\ldots = \int \frac{3}{4}\cdot \underbrace{4x}_{u'}\,(\underbrace{2x^2+1}_{u})^3\,dx = \frac{3}{4}\cdot\frac{(2x^2+1)^4}{4} + C$$

$$= \frac{3}{16}(2x^2+1)^4 + C$$

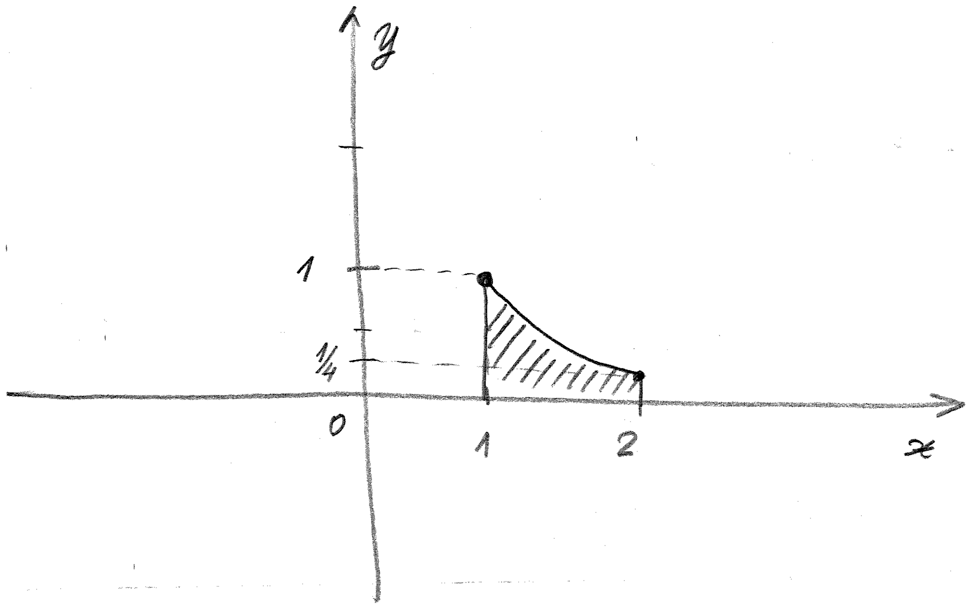
By substitution:

$$u = 2x^2+1 \Rightarrow du = 4x\,dx \Rightarrow x\,dx = \frac{du}{4}$$

$$\ldots = 3\int u^3\cdot\frac{du}{4} = \frac{3}{4}\int u^3\,du = \frac{3}{4}\,\frac{u^4}{4} + C = \frac{3}{16}u^4 + C$$

$$= \frac{3}{16}(2x^2+1)^4 + C$$

Problem 4. Use the definite integral to find the area of the region bounded by the graph of $y = \frac{1}{x^2}$, the x-axis and the vertical lines $x = 1$ and $x = 2$.



$$A = \int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_1^2$$

$$= -\frac{1}{2} + 1 = \frac{1}{2} \text{ units}^2$$

Problem 5. Use the definite integral to find the area of the region bounded by the curves $y = x^2$ and $y = 3x$.

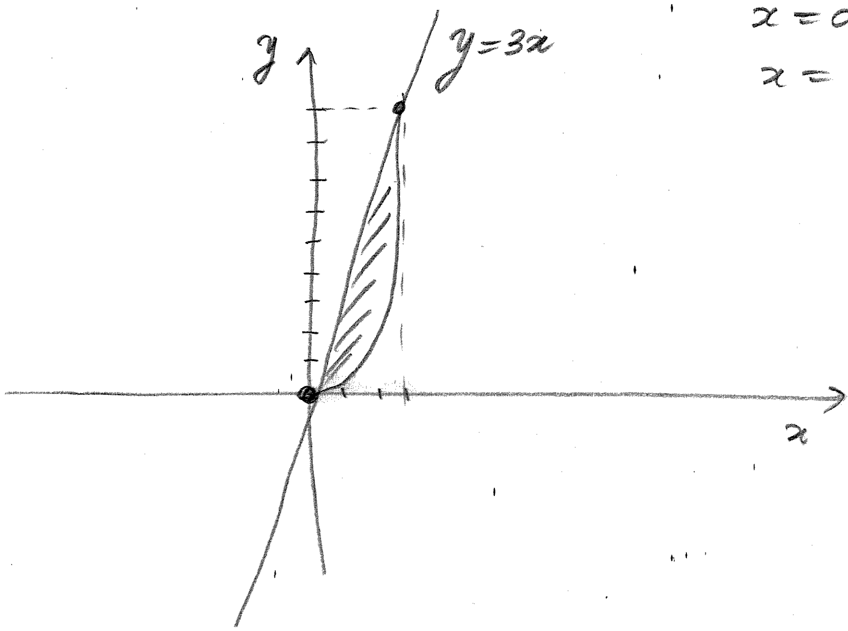
We get the intersection point first :

$x^2 = 3x \Rightarrow x = 0$ OR

$x = 3$

$x = 0 \Rightarrow (0,0)$

$x = 3 \Rightarrow (3,9)$



$$A = \int_0^3 3x - x^2 \, dx = \frac{3x^2}{2} - \frac{x^3}{3} \Big|_0^3$$

$$= \frac{3 \cdot 9}{2} - \frac{27}{3}$$

$$= 27\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ units}^2$$

Problem 6. Using the method of substitution evaluate the definite integral:

$$\int_0^1 \frac{x}{(x+1)^3}\,dx.$$

$$\boxed{\begin{aligned} u = x+1 \quad &\Rightarrow \quad x = u-1 \\ &\Rightarrow \quad du = dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=0 &\Rightarrow u=1 \\ x=1 &\Rightarrow u=2 \end{aligned}}$$

$$= \int_1^2 \frac{u-1}{u^3}\,du = \int_1^2 \frac{1}{u^2} - \frac{1}{u^3}\,du$$

$$= \int_1^2 u^{-2} - u^{-3}\,du$$

$$= \frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\Big|_1^2$$

$$= -\frac{1}{u} + \frac{1}{2u^2}\Big|_1^2$$

$$= -\cancel{\frac{1}{2}} + \frac{1}{8} + \cancel{\frac{1}{1}} - \cancel{\frac{1}{2}}$$

$$= \frac{1}{8}$$

Problem 7. Using the method integration by parts find the indefinite integral:

$$\int x^3 \ln x \, dx.$$

$u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x^3 dx \Rightarrow v = \frac{x^4}{4}$

$= (\ln x) \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^{\cancel{4}\,3}}{4} dx$

$= \frac{x^4}{4} \ln x - \frac{1}{4} \cdot \frac{x^4}{4} + C$

$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$

Problem 8. Find the indefinite integrals:

(a) $\int x^2 e^{x^3+1}\,dx.$ $= \frac{1}{3}\int 3x^2 e^{x^3+1}\,dx = \frac{1}{3}e^{x^3+1} + C$

(b) $\int \frac{x^2+1}{x^3+3x}\,dx.$ $= \frac{1}{3}\int \frac{3x^2+3}{x^3+3x}\,dx$

u'

u

$= \frac{1}{3}\ln|x^3+3a| + c$

$= \ln\sqrt[3]{|x^3+3a|} + c$

☐ **Problem 9.** A company expects its income c during the next 10 years to be modeled by $c(t) = 450$. Assuming an annual inflation rate of 4%, what is the present value of this income?

(Hint: You may use the formula of the Present value$=\int_0^{t_1} c(t)e^{-rt}\,dt$.)

$$\underline{\text{Present Value}} = \int_0^{10} 450\, e^{-0.04t}\, dt$$

$$= 450 \cdot \left. \frac{e^{-0.04t}}{-0.04} \right|_0^{10}$$

$$= \frac{450}{-0.04} \cdot \left(e^{-0.4} - 1 \right)$$

$$= \$\,3708.899482$$

Problem 10. Find the indefinite integrals:

(a) $\int \frac{1}{x^2+x}\,dx.$ $= \int \frac{1}{x} - \frac{1}{x+1}\,dx = \ldots$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$1 = A(x+1) + Bx$$

$$x = 0 \Rightarrow A = 1$$

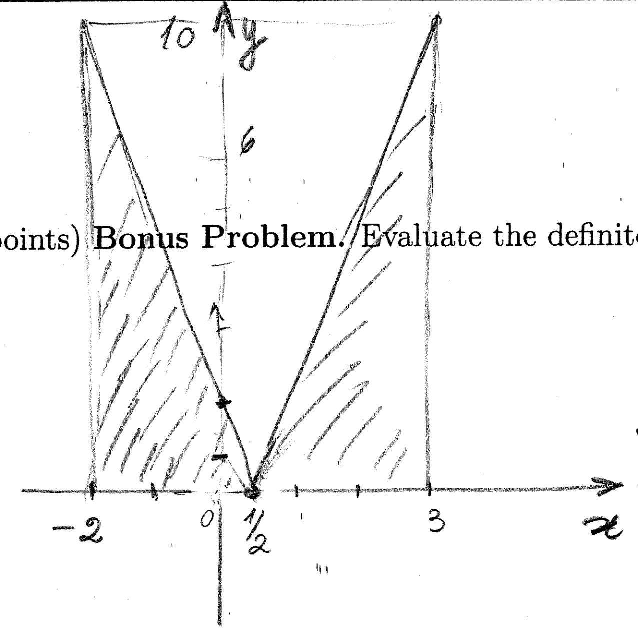
$$x = -1 \Rightarrow B = -1$$

$$\ldots = \ln|x| - \ln|x+1| + C = \ln\left|\frac{x}{x+1}\right| + C$$

(b) $\int \frac{x^3+x^2+1}{x^2+x}\,dx.$

$$\begin{array}{r|l} & x \\ \hline x^2+x & x^3+x^2+1 \\ & -(x^3+x^2) \\ \hline & 1 \end{array}$$

$$= \int x + \frac{1}{x^2+x}\,dx = \frac{x^2}{2} + \ln\left|\frac{x}{x+1}\right| + C$$



(5 points) **Bonus Problem.** Evaluate the definite integral $\int_{-2}^{3} |4x-2|\, dx$.

$4x-2=0 \Rightarrow x=\frac{1}{2}$

$|4(-2)-2| = |-10| = 10$

$|4\cdot 3-2| = 10$

$A = \frac{1}{2}\cdot 10\left(\frac{1}{2}+2\right) + \frac{1}{2}\left(3-\frac{1}{2}\right)\cdot 10$

$= \frac{25}{2} + \frac{25}{2} = 25 \text{ units}^2$

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Useful formulas

(1) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$.

(2) $\int \frac{1}{x}\, dx = \ln|x| + C$.

(3) $\int e^x\, dx = e^x + C$.

(4) The general power rule: $\int u^n \frac{du}{dx}\, dx = \frac{u^{n+1}}{n+1} + C$ for $n \neq -1$.

(5) $\int \frac{u'}{u}\, dx = \int \frac{1}{u}\frac{du}{dx}\, dx = \ln|u| + C$.

(6) $\int e^u \frac{du}{dx}\, dx = e^u + C$.

(7) Integration by parts: $\int u\, dv = uv - \int v\, du$.

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