Review problems for Final Exam

Math 1100, Fall 2005

(1) Find the derivative y' of the following functions:

(a) $y = \dfrac{5x}{3x^4 + 2}$.

(b) $y = (x^3 + 2x^2)^6 e^{3x}$.

(c) $y = \dfrac{3}{\sqrt{x}} - 7\ln(5x^2) + 4e^{8x}$.

(d) $y = \dfrac{1}{\sqrt{x}}\left(\sqrt[3]{x} - \dfrac{2}{x}\right)$.

(2) Find $f^{(4)}(x)$ if $f(x) = \dfrac{2}{x^3} + 5x^4 - 3x^2$.

(3) Find $\dfrac{dy}{dx}$ for the following functions:

(a) $y = x^2 + 3y^2 + 2x - 3y + 2$.

(b) $e^x y + x^2 = 4y$.

(4) Find the slope m of the tangent line of the function $f(x) = \dfrac{x}{(2x^2+1)^3}$ at the point $(0, 1)$.

(5) Find the length and the width of the rectangle with maximum area and with the perimeter of 120 meters.

(6) A farmer has 180 feet of fencing and wishes to construct three pens for his animals by first building a fence around a rectangular region, and then subdividing the region in three smaller rectangles y placing two fences parallel to one of the sides. What dimensions of the region will maximize the total area?

(7) Let $f(x) = \dfrac{2x^2 - 3x + 4}{(4x+3)(x-1)^2}$. Find the vertical and the horizontal asymptotes, if there are any.

(8) Find the relative minimum and maximum points of the function $f(x) = 2x^4 - 6x^2$.

(9) Find the inflection points of the function $f(x) = -x^3 + 3x^2 - 2$.

(10) Find the absolute maximum and the absolute minimum of the function $f(x) = \dfrac{1}{3}x^3 - 9x + 2$ on the closed interval $[0, 4]$.

(11) Sketch the graph of the function f having the given characteristics:

$$f'(-1) = f'(2) = 0,$$
$$f'(x) > 0 \text{ for } -1 < x < 0 \text{ and } 0 < x < 2,$$
$$f'(x) < 0 \text{ for } -\infty < x < -1 \text{ and } 2 < x < \infty$$
$$f'(0) \text{ does not exist}$$
$$f''(x) > 0 \text{ for } -\infty < x < 0$$
$$f''(x) < 0 \text{ for } 0 < x < \infty$$

(12) Find the future value of an \$20,000 investment if the interest rate is 4% compounded continuously for 10 years.

(13) Evaluate the integrals:

(a) $\displaystyle\int x^2\left(4x - \frac{3}{\sqrt{x}} + 2\right)\, dx.$

(b) $\displaystyle\int \frac{6}{2y-1}\, dy.$

(c) $\displaystyle\int 3x^2(3x^3+4)^3\, dx.$

(d) $\displaystyle\int_1^2 \frac{1}{(2x-1)^3}\, dx.$

(e) $\displaystyle\int \frac{1}{x\ln(2x)}\, dx.$

(f) $\displaystyle\int \frac{1}{x^2+3x+2}\, dx.$

(14) Evaluate the improper integrals:

(a) $\displaystyle\int_3^{\infty} \frac{2}{x^6}\, dx.$

(b) $\displaystyle\int_2^3 \frac{1}{\sqrt{x-2}}\, dx.$

(c) $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-x^3}\, dx.$

(15) Use the definite integral to find the area of the region bounded by $y = xe^{-x}$, $x = 0$, $x = \ln 2$ and $y = 0$.

(16) Factory orders for an air conditioner are about 6000 units per week when the price is \$325 and about 8000 units per week when the price is \$300. Find the consumer and the produces surpluses. (Assume that the demand function is linear.)

(17) Find the particular solution $y = f(x)$ that satisfies the differential equation $f'(x) = x^2\ln 2x$ and the initial condition $f(1) = 2$.

(18) You deposit an annuity of \$1000 in a saving account each year for 25 years at an interest rate of 4%. How much you will have in your account after 25 years ?

(19) Find the area of the region bounded by the graphs of $y = e^{-3x}$, $y = 0$, and $x \geq 0$.

(20) Find the domain and the range of the function $z = \dfrac{x^2+y^2}{\sqrt{x+y}}$.

(21) Let $f(x,y,z) = 4xyz + x^3y^2z + x^3$. Find the following:

(a) $\dfrac{\partial f}{\partial x}$.

(b) f_{xz}.

(c) f_{xyz}.

(d) The partial derivative of f with respect to y at the point $(1,2,0)$.

(22) Suppose that the total revenue function for a product is given by $R(x) = 44x - 0.02x^2$.

(a) How many units will maximize the total revenue?

(b) Find the maximum revenue.

(c) In the production is limited to 1000 units, how many units will maximize the total revenue?