Answers to review problems for Exam #2

Math 1100, Fall 2005

(1) (a) $\frac{d}{dx}\left[(3x^2+x)e^{-x^2}\right] = (-6x^3-2x^2+6x+1)e^{-x^2}$.

(b) $\frac{d}{dx}\left[(e^{2x}+1)^3\right] = 6e^{2x}(e^{2x}+1)^2$.

(c) $\frac{d}{dx}\left[\ln\sqrt{\frac{x-1}{x^2+1}}\right] = \frac{1}{2(x-1)} - \frac{x}{x^2+1}$.

(d) $\frac{d}{dx}\left[(\ln x^3)^2\right] = \frac{18\ln x}{x}$.

(e) $\frac{d}{dx}\left[\ln\frac{e^{x^2}}{1+e^x}\right] = 2x - \frac{e^x}{1+e^x}$.

(2) (a) $\frac{dy}{dx} = \frac{xe^{-x} - ye^x - 2y^2 - e^{-x}}{e^x + 4xy}$.

(b) $\frac{dy}{dx} = \frac{(3x-1+xy-2x^2)y}{(2-xy)x}$.

(3) (a) $f(x) = x^4 - 4x^2 + 4$.

(i) x intercepts: $(\sqrt{2},0)$ and $(-\sqrt{2},0)$; y-intercept: $(0,4)$.

(ii) V.A. none.

(iii) H.A. none. $\lim_{x\to\infty} x^4-4x^2+4 = \lim_{x\to-\infty} x^4-4x^2+4 = \infty$.

(iv) Relative minimum points: $(\sqrt{2},0)$ and $(-\sqrt{2},0)$. Relative maximum point: $(0,4)$.

(v) Absolute maximum point: $(5,529)$. Absolute minimum points: $(\sqrt{2},0)$ and $(-\sqrt{2},0)$.

(vi) Concave downward: $\left(-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}\right)$. Concave upward: $\left(-\infty,\sqrt{\frac{2}{3}}\right)$ and $\left(\sqrt{\frac{2}{3}},\infty\right)$.

(vii) Inflection points: $\left(-\sqrt{\frac{2}{3}},\frac{16}{9}\right)$ and $\left(\sqrt{\frac{2}{3}},\frac{16}{9}\right)$.

(b) $f(x) = \frac{2x^2-10}{x+3}$.

(i) x intercepts: $(\sqrt{5},0)$ and $(-\sqrt{5},0)$; y-intercept: $\left(0,-\frac{10}{3}\right)$.

(ii) V.A. $x=-3$, since $\lim_{x\to-3^+} f(x) = \infty$ and $\lim_{x\to-3^-} f(x) = -\infty$.

(iii) H.A. none, since $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.

(iv) Relative minimum point: $(-1,-4)$. Relative maximum point: $(-5,-20)$.

(v) Absolute maximum point: $(5,5)$. Absolute minimum point: $(-1,-4)$.

(vi) Concave downward: $(-\infty,-3)$. Concave upward: $(-3,\infty)$.

(vii) Inflection points: none.

(c) $f(x) = \frac{3x}{x+2}$.

(i) x-intercept: $(0,0)$; y-intercept: $(0,0)$.
(ii) V.A. $x=-2$, since $\lim_{x\to -2^+} f(x) = -\infty$ and $\lim_{x\to -2^-} f(x) = \infty$.
(iii) H.A. $y=3$, since $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$.
(iv) Relative minimum point: none. Relative maximum point: none.
(v) Absolute extrema: none.
(vi) Concave upward: $(-\infty,-2)$. Concave downward: $(-2,\infty)$.
(vii) Inflection points: none.

(d) $f(x) = \dfrac{4x}{9-x^2}$.
(i) x-intercept: $(0,0)$; y-intercept: $(0,0)$.
(ii) V.A. $x=-3$ and $x=3$, since $\lim_{x\to -3^+} f(x) = \infty$ and $\lim_{x\to -3^-} f(x) = -\infty$ respectively, $\lim_{x\to 3^+} f(x) = -\infty$ and $\lim_{x\to 3^-} f(x) = +\infty$.
(iii) H.A. $y=0$, since $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$.
(iv) Relative minimum point: none. Relative maximum point: none.
(v) Absolute extrema: none.
(vi) Concave downward: $(-3,0)$ and $(3,\infty)$. Concave upward: $(-\infty,-3)$ and $(0,3)$.
(vii) Inflection point: $(0,0)$.

(e) $f(x) = x^5 - 5x^4$.
(i) x-intercepts: $(0,0)$ and $(5,0)$; y-intercept: $(0,0)$.
(ii) V.A. none.
(iii) H.A. none, since $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.
(iv) Relative minimum point: $(4,-256)$. Relative maximum point: $(0,0)$.
(v) Absolute minimum point: $(4,-256)$. Absolute maximum points: $(0,0)$ and $(5,0)$.
(vi) Concave downward: $(-\infty,3)$. Concave upward: $(3,\infty)$.
(vii) Inflection point: $(2.4,-53.08416)$.

(f) $f(x) = x^2 e^{-x}$.
(i) x-intercept: $(0,0)$; y-intercept: $(0,0)$.
(ii) V.A. none.
(iii) H.A. $y=0$, since $\lim_{x\to\infty} f(x) = 0$. $\lim_{x\to-\infty} f(x) = -\infty$.
(iv) Relative minimum point: $(0,0)$. Relative maximum point: $\left(2, \dfrac{4}{e^2}\right)$.
(v) Absolute minimum point: $(0,0)$. Absolute maximum point: $\left(2, \dfrac{4}{e^2}\right)$.
(vi) Concave downward: $(2-\sqrt{2}, 2+\sqrt{2})$. Concave upward: $(-\infty, 2-\sqrt{2})$ and $(2+\sqrt{2},\infty)$.
(vii) Inflection points: $\left(2-\sqrt{2}, \dfrac{6-4\sqrt{2}}{e^{2-\sqrt{2}}}\right)$ and $\left(2+\sqrt{2}, \dfrac{6+4\sqrt{2}}{e^{2+\sqrt{2}}}\right)$.

(g) $f(x) = x\ln x$.
(i) x-intercepts: $(0,0)$ and $(1,0)$; y-intercept: none.
(ii) V.A. $x=0$, since $\lim_{x\to 0^+} f(x) = 0$. $\lim_{x\to 0^-} f(x) =$DNE.
(iii) H.A. none, since $\lim_{x\to\infty} f(x) = \infty$. $\lim_{x\to-\infty} f(x) =$DNE.
(iv) Relative minimum point: $\left(\dfrac{1}{e}, -\dfrac{1}{e}\right)$. Relative maximum point: none.
(v) Absolute extrema: none.

(vi) Concave upward: $(0, \infty)$.
(vii) Inflection points: none.

(4) To be done in class.

(5) The dimensions of the page are: 10.95×8.22 inches.

(6) \$330.

(7) 20 and 20.

(8) $16 \times 16 \times 32$ inches.

(9) (a) $\eta = -59$. Since $|\eta| = 59 > 1$, it is inelastic.
(b) $x = 100$ and $p = \$25$ for a revenue of \$2990.

(10) $P = \$8207.46$.

(11) $A(t) = 1000e^{0.03t}$.

(12) The cost of producing x units of a product is modeled by $C = 100 + 50x - 50 \ln x,\ x \geq 1$.
(a) $\overline{C} = \dfrac{100}{x} + 50 - \dfrac{50 \ln x}{x}$.
(b) The cost is minimum when $x = e^3$.