Review problems-Final exam

MATH 1210 - Fall 2004

1. (§2.2) If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{27 - x^3}$, find the domain of $g \circ f$.
2. (§2.4) Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x^2 - x}$.
3. (§2.7) Evaluate $\lim_{x \to 0^-} \frac{\sin |x|}{x}$.
4. (§9.1 + §2.7) If m and n are constants, evaluate $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$.
5. (§2.8) Evaluate $\lim_{x \to 0^+} x \cos\left(\frac{1}{x}\right)$.
6. (§2.8) Evaluate $\lim_{x \to \infty} \left(x - \sqrt{x^2 - 4x}\right)$.
7. (§2.8) Evaluate $\lim_{x \to \infty} \frac{\sqrt{x^2 + 4x}}{8x + 20}$.
8. (§2.8) Evaluate $\lim_{x \to 2^+} \frac{1 - x}{x^2 - 5x + 6}$.
9. (§2.9) The equation $x^3 - x - 5 = 0$ has one root in the interval $(-2, 2)$. This root is in the interval A. $(-2, -1)$ B. $(-1, 0)$ C.$(0, 1)$ D. $(1, 2)$ E. $(-1, 1)$.
10. (§2.9) If $h(x) = \begin{cases} x^2 + a, & \text{for } x < -1 \\ x^3 - 8, & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x.
11. (§3.2 + §3.3) If $f(x) = \frac{1}{x + 3}$, then evaluate $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$.
12. (§3.3) If $y = ax + b$ is the equation of the tangent line to the curve $y = x + \sin x$ at the point $x = \pi/2$, find a and b.
13. (§3.3) If the tangent line to the curve $y = 4 - 2x^2$ at the point where $x = a$ is parallel to the line $8x + 3y = 4$, then find a.
14. (§3.3) Find $f'(1)$ if $f(x) = \frac{1 - x}{1 + x}$.
15. (§3.3) Find $f'(x)$ if $f(x) = (x^2 - 1)\tan(x^2 + 1)$.
16. (§3.5) Find $\frac{d}{dx}\left(f(x^3)\right)$ if $f'(x) = \sqrt{x}$.
17. (§3.4 + §3.7) Find $f''(x)$ if $f(x) = \sin(x^2 + 1)$.
18. (§3.8) Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\frac{dy}{dx}$ at $(-2, 1)$.
19. (§3.9) A spherical balloon is being inflated in such a way that its radius increases at the rate of 1cm/s. In cm^3/s, how fast is the volume increasing 3 seconds after the inflation starts? (The volume of a spherical balloon of radius r is $4/3\pi r^3$ and r(0)=0).
20. (§3.9) The length of a rectangle is increasing at the rate of 2 m/s while its width is decreasing at 3 m/s. When the length is 6 m and the width is 4 m, find the rate of change of the area of the rectangle.
21. (§3.10) Use differentials to approximate $\sqrt{101}$.
22. (§4.1) Let f be a function whose derivative f' is given by $f'(x) = (x-1)^2(x-2)(x-5)$. Find the local maximum and local minimum points.
23. (§4.1) Find where the function $f(x) = \frac{2}{\sqrt{1 + x^2}}$ is decreasing.
24. (§4.1) For F a differentiable function on $(-\infty, \infty)$ and c a real number, which statement is true? I. If F has a local maximum at c, then $F'(c) = 0$. II. If $F'(c) = 0$, then F has a local maximum or minimum at c.

25. (§4.2) Which of the following is/are true about the function $g(x) = 4x^3 - 3x^4$? I. g is decreasing for $x > 1$. II. g has a relative extreme value at $(0, 0)$. III. The graph of g is concave up for all $x < 0$.
26. (§4.2) A function $f(x)$ satisfies $f''(x) = x^8 - x^2$. How many inflection points does $f(x)$ has ?
27. (§4.4) Find the maximum and minimum values of the function $f(x) = 3x^2 + 6x - 10$ on the interval $[-2, 2]$.
28. (§4.4) A rectangle is inscribed in the upper half of the circle $x^2+y^2 = a^2$ (one side is on the diameter). Calculate the area of the largest such rectangle.
29. (§4.6) Sketch a graph of a function f with the following properties: $f''(x) > 0$ for $x < 2$, $f'(2) = 0$ and $f'(x) < 0$ for $x > 2$.
30. (§5.1) Evaluate $\displaystyle\int 3t\sqrt[3]{2t^2 - 11}\ dt$.
31. (§5.2) If $f''(x) = 2 - \sin x$, $f(0) = 1$ and $f'(0) = 1$, find the value of $f(\pi)$.
32. (§5.2) Find the particular solution of the equation $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{y}}$ that satisfies the initial condition $y = 4$ at $x = 1$.
33. (§5.5) If $\displaystyle\int_2^3 f(x)\ dx = 3$ and $\displaystyle\int_5^2 f(x)\ dx = 4$, then find $\displaystyle\int_3^5 f(x)\ dx$.
34. (§5.5) Find $\displaystyle\int_1^3 f(x)\ dx$, if $f(x) = \begin{cases} 1, & \text{for } x \geq 2 \\ \dfrac{1}{x^2}, & \text{for } x < 2 \end{cases}$.
35. (§5.6) Find $\dfrac{d}{dx}\displaystyle\int_1^{2x} \sqrt{t^2 + 1}dt$ at $x = \sqrt{2}$.
36. (§5.6) If $F(x) = \displaystyle\int_x^{x^2} \sqrt{1 + t^3}\ dt$, then find $F'(1)$.
37. (§5.7) Write the limit $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + \frac{6i}{n} + \left(\frac{3i}{n}\right)^2\right] \frac{3}{n}$ as a definite integral.
38. (§5.8) Evaluate $\displaystyle\int_3^4 x\sqrt{25 - x^2}\ dx$.
39. (§5.8) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \sin x\ dx$.
40. (§6.1) Let R be the region between the graph of $y = 3x^2$ and the $x-$axis, from $x = a$ to $x = b$, $(0 < a < b)$. If the vertical line $x = c$ cuts R into two parts of equal area, then find c.
41. (§6.1) Write an integral that represents the area of the region R bounded by $y = -x + 2$ and $y = x^2$.
42. (§6.1) An object moves along a line so that its velocity at time t is $v(t) = 3t^2 - 6t - 24$ feet per second. Find the total distance traveled by the object for $-1 \leq t \leq 3$.
43. (§6.2 or §6.3) The region enclosed by the curves $y = x$ and $y = x^2$ is rotated about the y-axis. Find the volume of the resulting solid.
44. (§6.3) Let R be the region between $y = \dfrac{1}{x}$, $y = 0$, $x = 1$ and $x = 3$. Write an integral that represents the volume of the solid generated by revolving R about the line $x = 4$.
45. (§6.4) Write an integral that represents the length of the arc of the parabola $y = x^2$, $0 \leq x \leq \dfrac{1}{2}$.
46. (§6.5) If it takes 4 ft-lbs of work to stretch a spring from natural position to a distance 2 feet beyond, how much work is required to stretch the spring from 2 feet to 3 feet beyond the natural position?
47. (§6.5) A tank of height 10 feet and whose horizontal cross sections are squares of side 2 feet is filled with water. How much work is required to pump out all but 2 feet of water to the top of the tank? Water weights 62.5 lb/ft^3.
48. (§6.6) Find the centroid of the region bounded by the curves $y = 2x$, $y = 0$ and $x = 1$.