

Instructor: Oana Veliche

Time: 50 minutes

NAME: _____

ID#: _____

Problem	Max Points	Points
1	12	
2	15	
3	10	
4	12	
5	8	
6	10	
7	12	
8	12	
9	9	
Total	100	

Show work and justify all your answers.

(12 pts.) **Problem 1.** A rectangle is to be inscribed in a circle of radius 1 cm. What are the dimensions of the rectangle if its area is to be maximized?

Hint: First draw a circle in a xy coordinate system, centered at $(0, 0)$.

(15 pts.) **Problem 2.** Let

$$f(x) = \frac{x^2 + 2x + 1}{x}.$$

Using that the first and the second derivatives are respectively given by

$$f'(x) = \frac{(x-1)(x+1)}{x^2} \quad \text{and} \quad f''(x) = \frac{2}{x^3}$$

sketch the graph of the function f following the steps below:

(a) Find the domain, the x-intercept and the y-intercept of f .

(b) Find the horizontal, the vertical and the oblique asymptotes of f .

(c) Fill in the sign table, indicating the critical point(s), inflection point(s) and the intervals where f is increasing, decreasing, concave up and concave down.

x	
$f'(x)$	
$f''(x)$	
$f(x)$	

(d) Graph the function indicating on the graph the inflection point(s), local maxima, local minima, global maximum and global minimum, if they exist.

(10 pts.) **Problem 3.** Solve the differential equation:

$$\frac{dy}{dx} = \frac{\sqrt{x}}{y}$$

with the initial condition $y = 2$ at $x = 1$.

(12 pts.) **Problem 4.**

Compute the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x} =$$

$$(b) \lim_{x \rightarrow 0} \left(\frac{1}{|x|} - \frac{1}{x^2} \right) =$$

(8 pts.) **Problem 5.** Apply the Mean Value Theorem to the function

$$f(x) = \frac{x^2 + 1}{x}$$

on the interval $[1, 3]$. In case the theorem cannot be applied, explain why.

(10 pts.) **Problem 6.** Evaluate $\int_1^3 (x\sqrt{3} + \sqrt{5})dx$ using the definition of definite integral (Riemann Sum).

Hint: Use the formula $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ and $x_i = 1 + \frac{2i}{n}$.

(12 pts.) **Problem 7.** Find $G'(x)$, where $G(x) = \int_1^{\sqrt{x}} \sin\left(\frac{1}{t^2}\right) dt$.

(12 pts.) **Problem 8.** Evaluate the integrals:

(a) $\int_1^4 \frac{2x+1}{\sqrt{x}} dx =$

(b) $\int \sin(3x) \cos^2(3x) dx =$

(9 pts.) **Problem 9.** Circle the right answer and justify your choice:

(a) $\int_{-a}^a f(x) dx = 0$ for A. For f even. B. For f odd. C. For any f .

(b) $\int_a^b f(x) dx = -\int_b^a f(x) dx.$ A. True. B. False. C. Only for $a = b$.

(c) $0 \leq \int_1^2 1 + \cos(2x^2) dx \leq 2.$ A. True. B. False.