Sums

The story goes that Carl Gauss was misbehaving in his Kindergarten class one day, and to shut him up, his teacher told him to add all the numbers from 1 to 100! Gauss gave the answer in just a few seconds! How could he do it? Here is the picture:

Can you explain it and give a formula for the sum of numbers 1 to n?

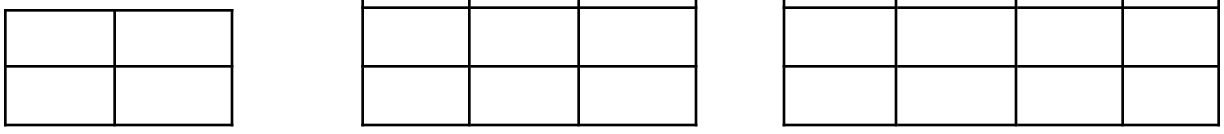
Magic Squares

A *magic square* is an $n \times n$ array of numbers, such that each row, each column, and the two diagonals add up to the same number. A *normal magic square* is one filled exactly with the numbers 1 up to n^2. We will just do normal magic squares here. No repeats!

1. What is the sum of the elements in a row of a 3 x 3 magic square? This number is called the *magic sum*.

2. What about for a 4 x 4 square? 5 x 5? A formula for the magic sum of an $n \times n$ square?

3. The smallest magic square is pretty silly: it is 1 x 1 and has the number 1 in it. Can you make the next biggest (2 x 2)? How about a 3 x 3? Harder: 4 x 4?



4. It turns out, that except for flipping or rotating, there is just one magic square that is 3 x 3. So you found it! What number is in the middle?

5. Hello again. You probably couldn't make a 2 x 2 square. Can you explain why not?

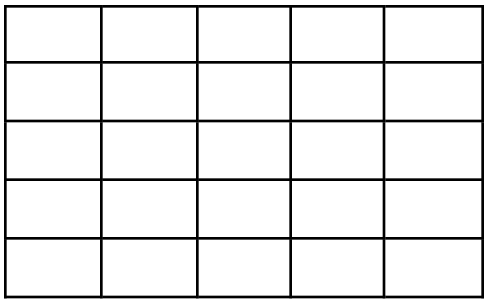
6. Oh, and that middle number on the 3 x 3. Was there anything special?

7. Let's test our understanding of directions. Here is how to make an *n* x *n* magic square for any odd *n*. Use it to fill in the 5 x 5 square below:

a) Put a 1 in the top square of the middle column.

b) Move up one and right one, wrapping around if you have to (for example, in a 3 x 3 square, the bottom right corner is "one to the right and one up" from the top middle), and put the next number in sequence here.

c) Repeat step b), placing 2,3, etc. until you hit a filled square. If going up one and right one puts you in a filled square, go down one instead, and start over.



What's that middle number again?

8. Just so you know: there are 880 4 x 4 magic squares. There are more than 275 million 5 x 5 magic squares. No one knows how many 6 x 6 magic squares there are.